CLASSICAL TESSELLATIONS AND 3-MANIFOLDS, SPRING 2014, MIDTERM

DANNY CALEGARI

This midterm exam was given out in class on Friday, May 2nd.

Problem 1. Let D_6 be the dihedral group with 12 elements. Give two *different* actions of D_6 on the sphere by isometries (i.e. such that the quotients give two different orbifolds).

Problem 2. Give an example of a group G that acts discretely by isometries on the plane, and which is torsion-free (i.e. every element except the identity has infinite order), but is *not* a group of translations. Give an example of a group G that acts discretely by isometries on 3-dimensional Euclidean space, and which is torsion-free and infinite, but in which *no* element (except the identity) is a translation.

Problem 3. Suppose Σ is a closed surface for which there is a nontrivial finite group G that acts on Σ in such a way that the quotient Σ/G is homeomorphic to Σ. What does this imply about the Euler characteristic of Σ? Give an example. Give an example where Σ is an *orbifold* (which is not a surface).

Problem 4. A *buckyball* is a tiling of the sphere by pentagons and hexagons, meeting 3 around every vertex. Thus a soccerball is an example of a buckyball. How many pentagons are there in a buckyball?

Bonus problem 5. How many hexagons can a buckyball have?

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CHICAGO, CHICAGO, ILLINOIS, 60637
E-mail address: dannyc@math.uchicago.edu