Math 27800 / CS 27800, Winter 2024: Problem Session 1

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Exercise 1. Prove that the following are well-defined operations in **ZFC**. First, write down precisely what this means in each case.

(a) $A \cup B$,

(b) $A \cap B$,

(c) $A \setminus B$.

Exercise 2. Recall Kuratowski's definition of ordered pair. Denote a pair as $\langle x, y \rangle$. Prove in **ZFC** that if $\langle x, y \rangle = \langle x', y' \rangle$ then $x = x'$ and $y = y'$. Take note of the axioms that you need to use to make this definition work.

Exercise 3. John's professor erased the definition from the board too quickly for him to write it down, so he had to jot it from memory. Instead of Kuratowski's definition, he wrote down: $\langle x, y \rangle = \{x, y\}$. What is wrong with this definition?

Rose suffered a similar issue, but instead she wrote: $\langle x, y \rangle = \{x, \{x, y\}\}$. Is there anything wrong with this definition?

Dave missed the class entirely, and came up with the following definition on his own: $\langle x, y \rangle = \{\{0, x\}, \{1, y\}\}$. Is there anything wrong with this definition?

Finally, Jade tried to simplify Dave's definition, and defined $\langle x, y \rangle = \{x, \{y\}\}$. What is wrong with this definition?

Bonus question: Can you come up with any interesting alternate definitions of your own?

Exercise 4. Given two sets A, B, define the cartesian product $A \times B$ and prove in **ZFC** that it exists.