Research Statement

1 Background and Introduction

As an algebraic topologist, I study conceptual approaches to concrete questions about the homotopy groups of spheres and manifold invariants. I am especially interested in the interaction between *higher structures* and *chromatic homotopy theory*.

For every prime p and *height* $n \geq 0$, there is a cohomology theory E_n called Morava E-theory. Goerss-Hopkins-Miller [GH04] built a commutative product and action of a certain group $\mathbb{G}_n$ on E_n. Chromatic theory, developed in work of Morava [Mor85], Miller-Ravenel-Wilson [MRW77], and Devinatz-Hopkins-Smith [DHS88; HS98; DH95], tells us that *understanding the fixed points* E_n^{hG}*, for* $G \subseteq \mathbb{G}_n$*, is a fundamental goal of modern homotopy theory.* Already when $n = 1$ these objects are extremely powerful: E_1 is p-adic K-theory, and $E_1^{hC_2}$ was used to study real vector bundles, resolve the Hopf invariant one problem [Ada60], and compute the image of J [Ada66; Qui71; Sul74]. At height 2, the spectrum $E_2^{hC_2}$ gave new bounds on immersions of real projective spaces [KW08], the spectrum $E_2^{hQ_8}$ is connected to modular forms [Hop94], and in fact the entire height 2 piece of the sphere is describable in terms of E_2^{hG} for various finite groups G [Goe+05; BG16]. The Hill-Hopkins-Ravenel solution to the Kervaire invariant one problem [HHR16], used in classifying exotic spheres [KM63], can be recast in terms of $E_4^{hC_8}$.

To study and utilize the spectra E_n^{hG} one often uses *genera*, which are cobordism invariants. At the prime 2, Ando-Hopkins-Rezk [AHR10] famously rigidified the Witten genus [Wit87] to a map $\mathsf{MString} \to E_2^{hQ_8}$. Recently, Hahn-Shi [HS17] produced a C_2-equivariant unitary genus $\mathsf{MU} \to E_n$ at all heights, where C_2 acts by complex conjugation on MU. This gives genera $\mathsf{MU}^{hC_2} \to E_n^{hC_2}$ on fixed points; work of Hill-Hopkins-Ravenel on MU^{hC_2} then leads to a complete understanding of $E_n^{hC_2}$ at the prime 2. The natural next step is to search for genera valued in $E_{k(p-1)}^{hC_p}$ at *odd* primes, or, better, C_p-equivariant genera valued in $E_{k(p-1)}$. Hill-Hopkins-Ravenel made the following conjecture describing what the source of these genera should be:

Conjecture (Hill-Hopkins-Ravenel)**.** There exists a C_p-spectrum MU_{μ_p}, non-equivariantly equivalent to $\mathsf{MU}^{\wedge(p-1)}$, and with Tate fixed points a module over ordinary mod p cohomology.

Much of my work over the past two years has centered around proving this conjecture. Jeremy Hahn and I have a candidate spectrum and are verifying the properties above. If successful, we could resolve the 3-primary Kervaire invariant one problem, obstruct immersions of lens spaces, and compute $\pi_* E_{k(p-1)}^{hC_p}$ for all k. We describe this work in §2.

To construct genera, a key idea has been to demand the genus to be *highly structured*. A *higher structure* is an algebraic structure where relations are witnessed by homotopies, and these witnesses are themselves subject to witnessed conditions, and so on. There is a hierarchy of multiplicative structures, due to Boardman-Vogt [BV73] and May [May72] : for $0 \leq k \leq \infty$, an $\mathbb{E}_k$-algebra is an object equipped with operations indexed by the configurations of points in $\mathbb{R}^k$. An $\mathbb{E}_1$-algebra is the higher version of an associative ring, an $\mathbb{E}_\infty$-algebra is the higher version of a commutative ring, and the others interpolate between. In our story, the Witten genus is an $\mathbb{E}_\infty$-map. I constructed a similar genus at the prime 3, answering a question of Hill-Lawson [HL13]:

Theorem (W.)**.** *There is an* $\mathbb{E}_\infty$ *map* $\mathsf{MSpin} \to E_2$ *at* $p = 3$ *which refines the Ochanine genus.*

The Hahn-Shi genus is a map of $\mathbb{E}_1$-rings with involution. Crucial to my work with Hahn is a new higher structure called a *spoke algebra*, which replace rings with involution for C_p. We discuss this in §2. The remaining sections detail related results and directions for future research.

2 Spoke Algebras, Koszul Duality, and Hochschild Cohomology

After localizing at a prime, MU has an indecomposable summand, BP, called the Brown-Peterson spectrum. It is enough for the applications to produce an equivariant analogue of this spectrum. To motivate our approach, we explain a new non-equivariant construction of BP as an $\mathbb{E}_1$-algebra.

Just as we use cohomology to study spectra, we use *topological Hochschild cohomology*, THC, to study $\mathbb{E}_1$-rings. If $A \to B$ is an $\mathbb{E}_1$-map, then $\mathsf{THC}(A; B)$ is the spectrum of A-bimodule maps $A \to B$. This object plays a key role in Koszul duality [Lur10] and deformation quantization [Kon99].

The spectrum $\mathsf{THC}(\mathbb{F}_p) := \mathsf{THC}(\mathbb{F}_p; \mathbb{F}_p)$ is much better behaved than the Steenrod algebra. Its coefficients are a commutative ring given by $\pi_*\mathsf{THC}(\mathbb{F}_p) = \mathbb{F}_p[y_0, y_1, ...]/(y_i^p)$, where $|y_i| = -2p^i$ [Bök]. The Deligne conjecture allows us to speak of $\mathbb{E}_1$-$\mathsf{THC}(\mathbb{F}_p)$-algebras, and Koszul duality implies that realizing $\mathsf{THC}(\mathsf{BP}; \mathbb{F}_p)$ as an $\mathbb{E}_1$-$\mathsf{THC}(\mathbb{F}_p)$-algebra is sufficient to build BP.

It is easy to construct the correct *module* over $\mathsf{THC}(\mathbb{F}_p)$ as the derived tensor (or smash) product:

$$A := \mathsf{THC}(\mathbb{F}_p) \otimes_{S^0[y_0,y_1,...]} S^0 =: \mathsf{THC}(\mathbb{F}_p)/(y_0, y_1, ...).$$

We were able to show that A admits an algebra structure as a special case of the following [HW18b]:

Theorem (Hahn-W.)**.** *Let R be an $\mathbb{E}_2$-ring with homotopy groups concentrated in even degrees, and $\{x_j\}$ a sequence of elements in $\pi_{2*}R$. Then $R/(x_1, x_2, ...)$ admits the structure of an $\mathbb{E}_1$-R-algebra.*

This generalizes results [Ang08; BSS16] with hypotheses not satisfied for $\mathsf{THC}(\mathbb{F}_p)$. We then prove:

Corollary (Hahn-W.)**.** *Let $A = \mathsf{THC}(\mathbb{F}_p)/(y_0, y_1, ...)$ and $B = A \otimes_{\mathsf{THC}(\mathbb{F}_p)} A$. Then the spectrum $\mathrm{Map}_B(A, \mathbb{F}_p)$ is equivalent to the p-completion of* BP.

To apply these ideas C_p-equivariantly, we must compute a version of $\mathsf{THC}(\mathbb{F}_p)$. Classically, one can do this using the Hopkins-Mahowald theorem characterizing $\mathsf{H}\mathbb{F}_p$ as an $\mathbb{E}_2$-algebra [MNN15]. Given any representation V of a group G, an $\mathbb{E}_V$-algebra has products indexed by configurations of points in V. These have been much studied in recent literature [BH13; GM17]. For example, if S^V denotes the compactification of V, then $\Omega^V X := \mathrm{map}_*(S^V, X)$ is canonically an $\mathbb{E}_V$-algebra. Using the Hopkins-Mahowald theorem and a *norm* operation we proved the following result [HW18a], where λ denotes $\mathbb{C}$ with its action by a p^kth root of unity:

Theorem (Hahn-W.)**.** *For all primes p, the C_{p^k}-spectrum $\mathsf{H}\mathbb{F}_p$ is the universal $\mathbb{E}_\lambda$-ring with $p = 0$.*

Let $\lambda/2 \subseteq \lambda$ be the union of rays through the p^kth roots of unity. Any $\mathbb{E}_\lambda$-algebra admits multiplications indexed by configurations of points in $\lambda/2$. We call such algebras $\mathbb{E}_{\lambda/2}$-algebras or *spoke algebras*. They play the role of 'algebras with involution' at odd primes. Non-equivariantly, spoke algebras arise as special inputs for the factorization homology of stratified manifolds [AFT17]. We are now developing the theory of spoke algebras, including a version of Hochschild cohomology, $\mathsf{THC}^{\lambda/2}$, and an obstruction theory which combines the approach in [HL17] with my work [Wil17b] on the slice filtration of [HHR16]. Recent developments [Bar+16] in indexed ∞-category theory allow us to translate arguments from [Lur17] to our setting without much pain.

Granting these results, we can prove the following equivariant analogues:

Claim (Hahn-W., in progress)**.** *There are generators $y_i : S^{2p^{i-1}\rho_{C_p}} \to \mathsf{THC}^{\lambda/2}(\mathbb{Z})$ such that $A := \mathsf{THC}^{\lambda/2}(\mathbb{Z})/(p, y_1, y_2, ...)$ admits the structure of an $\mathbb{E}_{\lambda/2}$-$\mathsf{THC}^{\lambda/2}(\mathbb{F}_p)$-algebra.*

Let B denote the p-fold tensor product of A with itself over $\mathsf{THC}^{\lambda/2}(\mathbb{F}_p)$, with C_p action given by cyclic permutation (an instance of the *norm* [HHR16]). Then B is an $\mathbb{E}_1$-ring that acts on A.

Claim (Hahn-W., in progress)**.** *The C_p-spectrum* $\mathrm{Map}_B(A, \mathbb{F}_p)$ *confirms the Hill-Hopkins-Ravenel conjecture; i.e., produces a model of (the p-completion of)* BP_{μ_p}*, and is the source of a spoke algebra genus valued in* $E_{k(p-1)}$.

We recall that this leads to a resolution of the 3-primary Kervaire invariant problem and to the computation of $\pi_* E_{k(p-1)}^{hC_p}$ for all k.

3 C_2 Revisited: Power Operations, Hochschild Homology, and 126-Manifolds

Before the above odd-primary program, I proved several new results at the prime 2 which suggest further avenues of study. For example, Priddy [Pri80] gave an elegant construction of BP using the knowledge of power operations on the dual Steenrod algebra. In [Wil16], I construct operations on the homology of $\mathbb{E}_{\infty\rho}$-spectra, where ρ is the regular representation, and compute their action on the C_2 dual Steenrod algebra of [HK01], generalizing a result of Steinberger [Bru+86]. This let me characterize BP without reference to complex conjugation or manifolds [Wil16]:

Theorem (W.)**.** *The C_2-spectrum* BP *is characterized by being indecomposable and weakly initial amongst unital C_2-spectra E with $E^0(S^{k\rho-1}) = 0$ for all k.*

Another important application, which motivated our work in [HW18a], was the following result proved with Mark Behrens [BW18]:

Theorem (Behrens-W.)**.** *The C_2-spectrum* $\mathsf{H}\mathbb{F}_2$ *is the universal $\mathbb{E}_\rho$-algebra where $2 = 0$.*

Behrens and his collaborators have used this result to begin investigating an equivariant approach to the telescope conjecture.

This result also gives a new computation of the Real Hochschild homology, THR, of $\mathbb{F}_2$. Real Hochschild homology was developed in [Dot+17] as a way to study the Hermitian K-theory of rings with involution. The integral variant Hahn and I prove in [HW18a] similarly implies a conjecture from [Dot+17], generalizing a theorem of Bökstedt [Bök]:

Theorem (Hahn-W.)**.** *As an equivariant* $\mathsf{H}\mathbb{Z}$*-module, we have:*

$$\mathsf{THR}(\mathbb{Z}) \cong \mathsf{H}\mathbb{Z} \oplus \bigoplus_{k \geq 1} \Sigma^{k\rho-1}\mathsf{H}\mathbb{Z}/k.$$

4 Odd Primary Steenrod Algebra

In my search for MU_{μ_p} I found some interesting phenomena in C_p-equivariant homotopy theory. In my thesis [Wil17a], I used work of Stojanoska [Sto12] to prove that the spectrum of level 2 modular forms [HL13] acts as a C_3-analogue of connective K-theory, a truncation of BP. Since $\mathsf{tmf}(2)$ detects higher height information than K-theory, this is an example of *height shifting*, a crucial computational benefit of equivariant approaches to chromatic homotopy theory.

Also in [Wil17a], I used the Segal conjecture to produce an equivariant refinement of the first Hopf invariant element. Let $S^{\lambda/2}$ denote the compactification of $\lambda/2$, and define $S^{k\lambda/2}$ similarly.

Theorem (W.)**.** *There is a map $\tilde{\alpha}_1 : S^{(2p-3)\lambda/2} \to S^0$ which refines the nonequivariant element $\alpha_1 \in \pi_{2p-3}S^0$ and such that the composite $S^0 \to S^{(2p-3)\lambda/2} \to S^0$ has degree p.*

The class $\tilde{\alpha}_1$ is detected by a cohomology operation, and serious computations of equivariant homotopy groups will require knowledge of the C_p-Steenrod algebra. Krishanu Sankar and I [SW] recently computed the additive structure of the dual Steenrod algebra by determining Steinberg summands of certain equivariant classifying spaces.

Theorem (Sankar-W.)**.** *$\mathsf{H}\mathbb{F}_p \wedge \mathsf{H}\mathbb{F}_p$ splits equivariantly as a direct sum with summands of the form $\mathsf{H}\mathbb{F} \wedge X$ where X is of the form S^V or $\Sigma^V C\theta$ for some V, and $\theta : S^\lambda \to S^2$ is the degree p cover.*

Unlike the case $p = 2$, $\mathsf{H}\mathbb{F}_p \wedge \mathsf{H}\mathbb{F}_p$ is *not* free as an $\mathsf{H}\mathbb{F}_p$-module when $p > 2$. This means that computations with the equivariant Adams spectral sequence will require methods similar to those used in [LM87] and more recently [Bea+17] to study the bo-Adams spectral sequence. At the prime two, an understanding of the equivariant or motivic Adams spectral sequence in a range leads to computations of *classical* homotopy groups in a much larger range (see [IX15]). Similar strides could be made at odd primes, and the result above is the first step.

We remark that each of the above results is predicted by the existence of MU_{μ_p}.

5 Future Direction: Equivariant Moduli Problems and Height Shifting

The construction of MU_{μ_p} outlined in §2 inspires many questions. For example:

Question. What are the C_p versions of complex orientations and formal group laws using MU_{μ_p}?

Araki [Ara79] and Hu-Kriz [HK01] answered these questions at the prime 2. At odd primes, we are investigating a theory which replaces $\mathbb{C}P^\infty$ by the C_p-space $(\mathbb{C}P^\infty)^{\times(p-1)}$. Our computations suggest that this theory produces a generalization of Drinfeld's formal $\mathbb{Z}_p[\zeta_p]$-modules [Dri74]. Ravenel first proposed [Rav84] that formal Drinfeld modules could aid computations because they exhibit *height shifting*: Drinfeld modules of low height can have underlying formal groups of high height. Unfortunately, there can be no direct analogue of the theory of Drinfeld modules in homotopy theory [Law07]. Equivariance gives an alternative route to height shifting phenomena.

In Lurie's work on elliptic cohomology, the $\mathbb{E}_\infty$-rings E_n arise as the solution to a moduli problem in spectral algebraic geometry. Lurie defines a notion of a *spectral p-divisible group*, produces universal spectral deformations $(R, \mathbb{G})$ of height n formal groups, and shows that Morava E-theory can be obtained by localizing $\mathsf{THH}_{\mathcal{O}_{\mathbb{G}}}(R)$, where $\mathcal{O}_{\mathbb{G}}$ is the $\mathbb{E}_\infty$-ring representing $\mathbb{G}$.

I make the following optimistic conjecture as a guide for my future research:

Conjecture. There is a notion of a *$\overline{\rho}$-formal group* over an $\mathbb{E}_{\infty\rho}$-ring, there are $\mathbb{E}_{\infty\rho}$-spectra E_{n,μ_p} with the underlying Bousfield class of $E_{n(p-1)}$, and these arise as localizations of $\lambda/2$-Hochschild homology of universal spectral deformation rings for this theory.

This conjecture is within reach for the group C_2, and is supported by computational evidence. The general case requires new ideas that I am excited to explore. This conjecture would imply the following generalization of Snaith's theorem [Sna81] which makes no mention of equivariance:

Conjecture. A localization of $\Sigma^\infty_+ (\mathbb{C}P^\infty)^{\times(p-1)}$ has the same Bousfield class as E_{p-1}.

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