ERGODIC PROPERTIES OF SKEW PRODUCTS IN INFINITE MEASURE

1. Abstract

Let (Ω, μ) be a shift of finite type with a Markov probability, and (Y, ν) a non-atomic standard measure space. For each symbol i of the symbolic space, let Φ_i be a measure-preserving automorphism of (Y, ν). We study skew products of the form $(\omega, y) \mapsto (\sigma\omega, \Phi_{\omega_0}(y))$, where σ is the shift map on (Ω, μ). We prove that, when the skew product is conservative, it is ergodic if and only if the Φ_i's have no common non-trivial invariant set.

In the second part we study the skew product when $\Omega = \{0, 1\}^{\mathbb{Z}}$, μ is a Bernoulli measure, and Φ_0, Φ_1 are $\mathbb{R}$-extensions of a same uniquely ergodic probability-preserving automorphism. We prove that, for a large class of roof functions, the skew product is rationally ergodic with return sequence asymptotic to $\sqrt{n}$, and its trajectories satisfy the central, functional central and local limit theorem. Joint work with Patricia Cirilo and Enrique Pujals.