Monotonicity and Concavity

HW 6, Due Wednesday 1/25

1. Understand the Monotonicity and Concavity Theorems and be able to apply them. Try the Concepts Review to section 4.2.

2. Since we will need to solve inequalities to apply the monotonicity and concavity theorems, it is worth looking back at our method for solving inequalities. In particular, explain how the IVT has been implicitly at work whenever we have used the method of split points.

3. Be able to analyze the monotonicity and concavity of functions as in 1-28 from section 4.2. Then find where the following functions are increasing, decreasing, concave up, and concave down.

   (a) $h(t) = 4t^3 - 3t^2 - 6t + 1$

   (b) $L(x) = 3x^5 - 5x^3 + 1$

   (c) $f(x) = \frac{x^2}{x^2+1}$

   (d) $g(\theta) = \sin^2\theta$ on $[0, 2\pi]$

   (e) $h(z) = z^2 - \frac{1}{z^2}$

4. (4.2, 30, 34) Sketch the graph of a continuous function on $[0, 6]$ satisfying the stated conditions. The conditions in (a) are for one function; the conditions in (b) are for a different function.

   (a) $f(0) = 8$; $f(6) = -2$; decreasing on $(0, 6)$; inflection point at the ordered pair $(2, 3)$; concave up on $(2, 6)$.

   (b) $f(0) = f(3) = 3$; $f(2) = 4$; $f(4) = 2$; $f(6) = 0$; $f'(x) > 0$ on $(0, 2)$; $f'(x) < 0$ on $(2, 4) \cup (4, 5)$; $f'(2) = f'(4) = 0$; $f'(x) = -1$ on $(5, 6)$; $f''(x) < 0$ on $(0, 3) \cup (4, 5)$; $f''(x) > 0$ on $(3, 4)$.

5. (4.2, 39) We proved the following inequalities last quarter using the properities of inequalities:

   (a) $x^2 < y^2$

   (b) $\sqrt{x} < \sqrt{y}$

   (c) $\frac{1}{x} > \frac{1}{y}$

   when $0 < x < y$. Now prove them using the Monotonicity Theorem instead.

6. (4.2, 37) Prove that if $f'(x)$ exists and is continuous on an interval I and if $f'(x) \neq 0$ at all interior points of I, then either f is increasing throughout the interval I or decreasing throughout I. Hint: IVT.