Week 9, Due Wed 12/06

1. Prove that there do not exist any simple groups of the following orders. (Warning: not in order of difficulty)
   (a) 30
   (b) 72
   (c) 90
   (d) 112
   (e) 120
   (f) 126
   (g) 132
   (h) 140
   (i) 144
   (j) 150
   (k) 156
   (l) 200
   (m) 300
   (n) (*) 336
   (o) 1176
   (p) 2907
   (q) 6545
2. Suppose that G is a p-group, and $H \subset G$ has index p. Prove that H is normal in G.
3. Let N be a normal subgroup of G, and suppose that the largest power of p dividing $|N|$ is equal to the largest power of p dividing $|G|$. Prove that the p-Sylow subgroups of G are precisely the p-Sylow subgroups of N.
4. (*) We previously constructed groups $G = \mathrm{PGL}_2(\mathbf{F}_9)$ and $H = \mathrm{PSL}_2(\mathbf{F}_9)$ of orders 720 and 360.
   (a) Prove that $H \simeq A_6$.
   (b) Prove that H is a normal subgroup of G.
   (c) If $g \in G \setminus H$, show that conjugation by g on H is an automorphism which is not inner.
   (d) Prove that G is *not* isomorphic to S_6.