

Math 151 Section 33

Second Midterm

November 15, 2006

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Name:

This test is worth 100 points; the value of each question is provided. Please show your work so that partial credit can be awarded.

1. (5 points each)

(a) $f'(x)$, where $f(x) = 6x^2 - 3x + 3$.

(b) $\frac{d}{dx}(x \tan(x))$

(c) Find $f'(0)$, where

$$f(x) = \begin{cases} (x - 1)^2 & \text{if } x < 0 \\ (x + 1)^2 & \text{if } x \geq 0 \end{cases}$$

2. (10 points each)

(a) Find an equation for the tangent line to the ellipse $\frac{x^2}{9} + \frac{y^2}{36} = 1$ at the point $(-1, 4\sqrt{2})$.

(b) Find $\frac{d^2y}{dx^2}$, where $\cos(xy^2) = 1$.

3. (a) (5 points) Write the mathematical definition for $f'(x)$.

(b) (10 points) Use the above definition (and any applicable limit rules from chapter 2) to prove that for any real number c and any differentiable function f ,

$$(c \cdot f)'(x) = c \cdot f'(x).$$

4. (10 points) Suppose f is continuous at some point x . Must f be differentiable at x ? If so, prove it; if not, provide a counterexample and show why it is a counterexample.

5. (8 points) Let $y = \frac{u+1}{u+4}$ and $u = \frac{1}{x}$. Find $\frac{dy}{dx}$.

6. Oh no! Pirate Jim was dancing on the deck of his pirate ship, which is 19.6 meters above water level, when suddenly he looked down and realized he had danced right off the edge of his ship! He froze for an instant, cartoon-style, and then fell into the water.

Fact: When one falls cartoon-style, the gravitational pull is a constant 9.8 m/s^2 downward and there is no air resistance.

(a) (3 points) Set up a function $y(t)$ giving Pirate Jim's height above the water at time t .

(b) (5 points) After how many seconds did Pirate Jim hit the water? (Ouch!)

(c) (5 points) What was his speed at this instant?

(d) (3 points) What was his average velocity over the interval of time between when he began falling and when he hit the water?

7. (16 points) Pirate Jim has an unusually-shaped ship: it is a cone (tip-down) which is 30 meters deep and 40 meters across. He is 3 meters tall and standing at the tip (that is, the bottom). Suddenly (oh no!) his ship springs a leak which lets water in at a rate of $\frac{1}{2}$ cubic meters per second. How fast is the water level rising at the instant it reaches the top of his head? (Assume Pirate Jim is so thin that his volume is 0.)