Schrödinger type equations, Talbot's effect, and Riemann's "non-differentiable" function

Konstantin I. Oskolkov

Department of Mathematics, University of South Carolina
Columbia, SC 29208
e-mail: oskolkov@math.sc.edu

Fractal properties will be discussed, inherent in the solutions of the Cauchy initial value problem posed for Schrödinger type equations with the periodic initial data

$$\partial_t \psi = P(\partial_x)\psi, \quad \psi(t,x)|_{t=0} = f(x) \sim \sum_n \widehat{f}_n e^{2\pi i n x} \,.$$

Here, ∂_t, ∂_x denote the Schrödinger differential operators, e. g. $\partial_t := \frac{1}{i}\frac{\partial}{\partial t}$, and P is an algebraic polynomial with the real coefficients.

These properties, in particular, include the *Talbot's effect* of optical diffraction on the periodic grating. They are also directly related with differentiability (and non-differentiability) of functions of the type

$$F_r(t,x) := \sum_{n \neq 0} \frac{e^{\pi i (t n^r + 2nx)}}{\pi i n^r} \,, \quad r \in \mathbb{N}, \quad (t,x) \in \mathbb{R}^2.$$

For $r = 2$, the function

$$R(t) := \Re F_2(t,0) = \frac{2}{\pi} \sum_1^\infty \frac{\sin \pi n^2 t}{n^2}$$

was proposed by B. Riemann as a plausible example of a continuous and nowhere differentiable function.