

Math 312, Autumn 2007

Problem Set 5

Rudin, Chapter 5: 4, 5, 6, 8, 9, 11, 15, 18, 22

Probability Notes: Exercise 5.7, 5.8

Exercise 1 *Suppose H is a Hilbert space with a countable basis $\{u_1, u_2, \dots\}$. Define $\Lambda : H \rightarrow H$ to be the linear function with*

$$\Lambda(u_n) = n^{-1}u_n, \quad n = 1, 2, \dots$$

- *Show that Λ is bounded and find $\|\Lambda\|$.*
- *Let $U = \{v \in H : \|v\| < 1\}$. Show that ΛU does not contain δU for any $\delta > 0$.*
- *Why does this not contradict the open mapping theorem?*