Math 270, Spring 2008

Problem Set 8

Reading: Chapter VIII
Problems: p. 218, 6; p. 250, 6,7,9,12; p. 255: 1,3,4; p. 270, 2

Extra Problems

Exercise 1 *For which* $\alpha = \alpha(d)$ *is* $f(x) = |x|^\alpha$ *a harmonic function in* $\mathbb{R}^d \setminus \{0\}$*?*

Exercise 2 *Prove the following version of* Harnack's inequality*. For every* $r < 1$*, there is a* $c_r < \infty$ *such that the following is true. Suppose* u *is a harmonic function in the open unit disk in* $\mathbb{C}$ *taking values in* $(0, \infty)$*. Then for every* $|z| \leq r$*,* $u(z) \leq c_r\, u(0)$*.*

Exercise 3 *Suppose* C *is a closed, simple curve in* $\mathbb{C}$ *oriented counterclockwise surrounding a bounded open set* U*. Suppose* u *is a harmonic function on an open set* V *that contains* $\overline{U}$*. Show that*

$$\int_C \frac{du}{dn}(z)\, |dz| = 0.$$

Here n *denotes the unit normal to* C *pointing into* U *and* $|dz|$ *denotes an integral with respect to arc length. (You may wish to convert this integral into a line integral from multivariable calculus.)*