Math 270, Spring 2008

Problem Set 8

Reading: Chapter VIII
Problems: p. 218, 6; p. 250, 6,7,9,12; p. 255: 1,3,4; p. 270, 2

Extra Problems

Exercise 1 For which $\alpha = \alpha(d)$ is $f(x) = |x|^\alpha$ a harmonic function in $\mathbb{R}^d \setminus \{0\}$?

Exercise 2 Prove the following version of Harnack’s inequality. For every $r < 1$, there is a $c_r < \infty$ such that the following is true. Suppose $u$ is a harmonic function in the open unit disk in $\mathbb{C}$ taking values in $(0, \infty)$. Then for every $|z| \leq r$, $u(z) \leq c_r u(0)$.

Exercise 3 Suppose $C$ is a closed, simple curve in $\mathbb{C}$ oriented counterclockwise surrounding a bounded open set $U$. Suppose $u$ is a harmonic function on an open set $V$ that contains $\overline{U}$. Show that

$$\int_C \frac{du}{dn}(z) |dz| = 0.$$ 

Here $n$ denotes the unit normal to $C$ pointing into $U$ and $|dz|$ denotes an integral with respect to arc length. (You may wish to convert this integral into a line integral from multivariable calculus.)