STAT 25100
Spring Quarter 2007 ("Second Half")
Greg Lawler, 415 Eckhart
Office Hours: Tuesday, 4:30 – 5:30, Wed 4:45 – 5:45 and by appointment.

Reading in Ross
May 3: 5.1 – 5.3
May 8: 5.4 – 5.5
May 10: 5.6–5.7, 10.2
May 15: 6.1 – 6.3
May 17: 6.4 – 6.6
May 22: 7.5 – 7.6
May 24: 7.7 – 7.8
May 29: 8.3
May 31: no new material

HW 5 (due May 10)

1. (2pt) Use the Poisson approximation for the binomial to estimate the probability that in 10,000 (5 - card) poker hands that a player gets a flush at least 10 times. (A poker hand is a flush if all five cards are of the same suit.)

2. (2pt) (Ross, p.199, #19) Show that if X is a Poisson random varible with parameter λ and n is a positive integer,

$$E[X^n] = \lambda\, E[(X+1)^{n-1}].$$

Use this to compute $E[X^3]$.

4. (2pt) A continuous random variable X has a pdf of the form

$$f(x) = \begin{cases} cx^3, & 0 < x < 1, \\ c_1 x^2, & -1 < x < 0, \end{cases}$$

where c, c_1 are constants. It is known that $E[X] = 0$. What are c, c_1?

5. (3pt) A point is chosen at random from the semicircle $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1, y \geq 0\}$. To be precise, a number θ is chosen from the uniform distribution on $(0,\pi)$ and the point chosen is $(\cos\theta, \sin\theta)$. Let Y be the distance from the point to the x-axis. Find: the cdf of Y, the pdf of Y and $E[Y]$.

6. (3pt) (based on Ross, p. 248, #14). Let X be a uniform $(0,1)$ random variable and $n > 0$.

- Find the pdf for random variable X^n.
- Use this to compute $E[X^n]$.
- Check this result by using the quicker formula (Proposition 2.1 in Ross).

7. (2pt) (based on Ross, p. 318, #46) A complex machine is able to operate effectively as long as at least 3 of its 5 motors are functioning. Let T be the time until the machine stops working. If each motor independently functions for a random amount of time with density $e^{-x}, x > 0$, find the cdf for the random variable T.

8. (2pt) A point X is chosen from the uniform distribution on $(0, 1)$. After X is chosen a point Y is chosen from the uniform distribution on X. Find the cdf and the pdf for the random variable Y.

9. (2pt) Imagine the following experiment. First, one flips a coin two times. Let X be the number of heads in the two flips. Next, we let Y be a Poisson random variable with parameter X. In other words Y is a Poisson random variable with a random parameter. Does Y have a Poisson distribution? If yes, what is the intensity? If no, why not?

10. *(4pt) Suppose you play a game many times. Each time you play you have probability 1/2 of winning and 1/2 of losing. If you play the game $2n$ times, then your expected number of wins is n. Let p_n be the probability that in $2n$ games you win *exactly* n times. We are interested in

$$\lim_{n\to\infty} n^{1/2} p_n.$$

- Let $b_n = \sqrt{n}\, p_n$. Write an equation for b_n in terms of b_{n-1}
- Use this equation and a computer to make a guess for the limit.
- Use this equation to prove that the limit exists (even if you cannot show what the value of the limit is).