STAT 25100
HW 6 (due May 17)

1. (2pt) Use the normal approximation to the binomial to estimate the following probabilities

- The probability that in 400 rolls of a die one gets "6" between 75 and 90 times, inclusive
- Assuming that 30% of the population generally approves of the performance of the president, that in a town meeting of 500 randomly selected from the population, that at least half of the people would be supporters of the president.

2. (2pt) (Ross, p. 249 #22) The width of a slot of a duralamin forging is (in inches) normally distributed with $\mu = .9000$ and $\sigma = .0030$. The specification limits were given as $.9000 \pm .0050$.

- What percentage of forgings will be defective?
- What is the maximum allowable value of σ that will permit no more than 1 in 100 defectives when the widths are normally distributed with $\mu = .9000$ and σ?
- (optional) What is a duralumin forging and who cares?

3. (2pt) (based on Ross, p.250, #32) The time (in hours) required to replace a machine is an exponentially distributed random variable with parameter $\lambda = 1/3$. What is

1. The probability that the repair time exceeds 2 hours?
2. The probability that the repair time is between one and three hours?
3. Given that the repair has taken at least 10 hours, what is the probability it will take at least another 9 hours?

4. (2pt) Which of the following situations is modeled well by an exponential distribution? Give reasons.

- The lifetime of a lightbulb.
- The lifetime of a human.
- For a casual fisherman on a Saturday morning — the amount of time until the first fish is caught.

5. (4pt) (based on Ross, p.248, #13) You arrive at a bus stop at 10:00 knowing that on the average there are two buses every hour. Consider two possibilities:

- The time until arrival of the next bus is uniform on (0,30) (in minutes) This would be reasonable, say, if you knew buses came exactly every half hour but didn't know when this occured.
- The time until arrival is exponential with parameter 30

Answer these questions

- What is the expected amount of time you have to wait?
- What is the probability that you will have to wait longer than 10 minutes?
- If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

Do you think the uniform or the exponential is a better model?

6. (2pt) Suppose the number of customers arriving at a store is modeled by a Poisson process with intensity $\lambda = 5$. Here X_t is the number of customers who have arrived by time t (t measured in hours).

- Find the probability that the second customer arrives some time in the second hour.
- Find the conditional probability that that the second customer arrives in the second hour given that the number of customers in the first hour is zero or one.

7. (2pt) Suppose X has a standard normal distribution and $Y = e^X$. Find the pdf for Y.

8. (2pt) Suppose $X_1, X_2, \ldots, X_n$ are independent random variables, each $U(0,1)$. Suppose the numbers are ordered $Y_1 < Y_2 < \cdots < Y_n$. Find the cdf and pdf for Y_k and identify it as one of the distributions from Chapter 5.

9. (2pt) X is a Cauchy random variable if it has pdf

$$f(x) = \frac{c}{1+x^2}, \quad -\infty < x < \infty,$$

where c is a constant.

- What is c?
- Suppose X is a Cauchy random variable. What is the pdf for $Y = 1/X$?
- Does $E(X)$ exist, and, if so, what does it equal?

10. *(4pt) This is a computer exercise. Find computer software that includes a random number generator that produces uniform numbers between zero and one. Do the following many times:

- Take independent exponential random variables each with intensity $\lambda = 1$ and see how many you need until the sum of the random variables is at least 4. Record the number needed, say N, and subtract 1.
- Do this a large number of times (say at least 10,000) and give estimates for the probability that the number $N - 1$ equals $0, 1, 2, 3, \ldots$.
- What should the probabilities for N be? Compare the results from your simulation to this. (You may answer this question even if you do not want to do a simulation and receive one point.)