STAT 25100

GRADUATING SENIORS: Final Exam, Wednesday, May 30, 7-9 pm. More details later.

May 21 quiz: Sections 6.1–6.5 of Ross primarily.

HW 7 (due May 24)

1. (3pt, Ross p.314 #9). The joint pdf of (X, Y) is given by

$$f(x, y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \quad 0 < x < 1, \quad 0 < y < 2.$$

- Verify that this is a joint pdf.
- Find the marginal density for X.
- Find $\mathbf{P}\{Y > \frac{1}{2} \mid X < \frac{1}{2}\}$
- Find $E[X]$ and $E[Y]$.
- Find the conditional density $f_{Y|X}(y \mid x)$.

2. (3pt) Suppose X, Y are independent random varaibles where $X \sim U(0, 1)$ and Y is exponential with parameter 1. Find the pdf for the random variables $Z = X + Y$ and $W = X/Y$. Use the convolution method for Z and the cdf method for W.

3. (2pt) Let $X_1, \ldots, X_n$ be independent exponential random varaibles with parameter λ. Find the pdf for $\min\{X_1, \ldots, X_n\}$ and $\max\{X_1, \ldots, X_n\}$. Is either of these exponentially distributed? If so, with what parameter?

4. (2pt, Ross p. 317, #33) Jill's bowling scores are (approximately) normally distributed with mean 170 and standard deviation 20 and Jack's scores are approximately normal with mean 160 and standard deviation 15. If Jack and Jill each bowl a game and their scores are independent, find:

- The probability that Jack's score is higher.
- The total of their scores is above 350.

For the purposes of this problem, you may assume (although it is incorrect) that bowling scores are continuous random variables.

5. (2pt, Ross p. 321, #17) Let X_1, X_2, X_3 be independent, identically distributed continuous random variables. Compute

$$\mathbf{P}\{X_1 > X_2 \mid X_1 > X_3\},$$
$$\mathbf{P}\{X_1 > X_2 \mid X_1 < X_3\},$$
$$\mathbf{P}\{X_1 > X_2 \mid X_2 > X_3\}.$$

- Note that the problem did not specify the distribution of the random variable. Why are your answers true for any continuous distribution?
- Are your answers still true if the word continuous is replaced with discrete?

6. *(2pt) True or false: If X, Y, Z are random variables and X is independent of Y and X is independent of Z, then X is independent of $Y + Z$. If you say true, give a proof. If you say false, give an example where this does not hold.

7. (4pt) A random variable is said to have a *lognormal distribution* with parameters μ and σ^2 if $\log(X)$ has a normal distribution with mean μ and variance σ^2. (Here log refers to the natural logarithm.)

1. Find the pdf f for X,
2. Find the median for f, i.e, the number c such that $\mathbf{P}\{X \leq c\} = 1/2$.
3. Find the mode for f, i.e., the number c that maximizes $f(c)$.
4. Suppose r_1, r_2, r_3 are real numbers and X_1, X_2, X_3 are independent $N(0,1)$ random variables. Let $Y = X_1^{r_1} X_2^{r_2} X_3^{r_3}$. Show that Y has a log normal distribution and give its parameters.
5. Suppose that $\mu_1 = 0, \mu_2 = 2, \mu_3 = 3, \sigma_1 = 2, \sigma_2 = 1, \sigma_3 = 3$. Find the probability that the polynomial
$$X_1 t^2 + X_2 t + X_3 = 0$$
has real roots.

8. *(2pt) Suppose that X_1, X_2, X_3 are independent random variables each $U(0,3)$. Let
$$Y = \min\{|X_1 - X_2|, |X_1 - X_3|, |X_2 - X_3|\}.$$
Find the cdf and pdf for Y.

9. (2pt) Suppose that X, Y are independent $U(0,1)$ random variables and $Z = XY$. Find the joint pdf $f_{X,Z}(x,z)$ for (X, Z).

10. (2pt) Use the convolution method to determine the density for $X_1 + X_2 + X_3 + X_4$ where $X_1, \ldots, X_4$ are independent $U(0,1)$. (Hint: we computed the density for $X_1 + X_2$ in class.)