

Math 16200 Homework 6.

Problem 1 Using the product rule and the fact that $g(x) * \frac{1}{g(x)} = 1$ find the derivative of $\frac{f(x)}{g(x)}$ at a point a where $g(a) \neq 0$.

Problem 2

a) For natural numbers n show that the derivative of $f(x) = x^n$ is nx^{n-1} . One way to do this is by mathematical induction.

b) Prove the same formula when n is a negative integer.

Problem 3. Give an ϵ, δ proof that the function $f(x)$ defined on $[-1, 1]$ by $f(x) = x^2 \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$ is continuous at 0.

Problem 4 Show that the function define in problem 3) above is differentiable for all x (including 0) but $f'(x)$ is not continuous at $x = 0$.

(You may use what you know about the derivative of the sin function.)