MATH 16200 SECTION 30, HOMEWORK 3

DUE DATE TUESDAY, JAN 27

(1) Prove Lemma B, part (3).
(2) Prove Lemma C, part (2).
(3) Let $f, g\colon F \to F$ be continuous.

(a) Define a function $f+g\colon F \to F$ by $(f+g)(x) = f(x)+g(x)$. Show that $f+g$ is continuous.
(b) Define $f \circ g\colon F \to F$ by $(f \circ g)(x) = f(g(x))$. Show that $f \circ g$ is continuous.

(It is also true that products and quotients of continuous functions are continuous. If you're bored, feel free to prove this for yourself. This implies that all polynomials and rational functions are continuous on their entire domains.)

(4) Let $A \subset C$ and let $f : A \to C$ be continuous. Given $\lambda \in C$, which of the following sets are closed or open relative to A? Give brief proofs of your answers.
(a) $\{x \in A;\ f(x) < \lambda\}$,
(b) $\{x \in A;\ f(x) \leq \lambda\}$,
(c) $\{x \in A;\ f(x) \geq \lambda\}$,
(d) $\{x \in A;\ f(x) > \lambda\}$,
(e) $\{x \in A;\ f(x) = \lambda\}$.

(5) Let $f\colon A \to B$ be a function, and let x be a point in A which is not a limit point of the set A. Prove that f is continuous at x.

(6) Let $f\colon [a,b] \to \mathbb{R}$ and let $|f|$ be the function defined by $|f|(x) = |f(x)|$. For each of the following statements, either prove it or give a counterexample.
(a) If f is continuous, then $|f|$ is continuous.
(b) If $|f|$ is continuous, then f is continuous.