

Math 327, Spring 2009

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Course web page
<http://www.math.uchicago.edu/~may/327.html>

Class: MWF, 1:30 – 2:45
(That is an estimated stopping time, like an airplane’s estimated time of arrival.)

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In roughly their order of appearance, here are some sources. The book that comes closest to covering all that I would like to cover is Weibel’s, and Laurie Wail has ordered copies for you.

- S. Mac Lane. Categories for the working mathematician.
- F. Borceux. Handbook of categorical algebra 1.
- S. Mac Lane. Homology.
- C.A. Weibel. An introduction to homological algebra.
- H. Matsumura. Commutative ring theory.
- D. Eisenbud. Commutative algebra.

I will assume that you know the things in Atiyah-MacDonald. That 1969 book was written from a classical point of view, “stopping short of any results requiring a deep study of homological algebra”. At the time, the fundamental results relating homology to the structure theory of rings were still quite new. Of course, in one quarter we won’t go very deeply either, and we will probably just barely get to derived and triangulated categories. Our main focus will be on the basics of homological algebra, with applications to the study of the structure theory of commutative local rings and other kinds of rings and algebras. A more detailed tentative outline is on the next page.

Homework: An assignment will be given every week. Problems are to be turned in one week from the time they are handed out. While late problems will be accepted, they may not receive full credit.

Problems for the first five weeks have been posted on the course web site:

<http://www.math.uchicago.edu/~may/327.html>

I’m assuming that the problem sets do not have to be printed out for you.

There are no exams. No R's will be given to people who do most of the homework and get most of it right in a reasonably timely fashion. C's will be given if there is serious lack of understanding or if a significant portion of the homework is not done or is usually done late. *Late packets of homework are unfair to the grader.* As a rough guideline, based on past experience, I expect people to complete, accurately, at least 75% of the homework problems. The more the better. But the problems are intended to be fun: do what you can, and don't agonize over them. Work together but write up the problems yourself.

This will be an experimental course that mixes a range of topics focused largely on homological algebra. Possible topics, by week, include the following. I'm aware that this is way too ambitious, but I'm hoping you will agree that it is worthwhile to see a range of topics rather than focus too much on any one.

- (1) Categorical language; limits, colimits, and universal properties. Isomorphism and equivalence of categories. A bit on Abelian, monoidal, and enriched categories; maybe bicategories.
- (2) Basic homological algebra: Projectives (which you have seen) and injectives (which you may not have seen), axioms for Tor and Ext, construction and verification of the axioms, homological dimension, and examples.
- (3) Dedekind rings and their characterizations; number rings.
- (4–5) Some local ring theory and dimension theory: Cohen-Macaulay rings, regular local rings, and their homological characterization, following up from where the winter quarter course left off.
- (6–8) Some basic Lie algebra theory and a bit on restricted Lie algebras. An introduction to the homology of groups and its relationship to topology. An introduction to the homology of Lie algebras. Algebras, coalgebras, bialgebras, and Hopf algebras; the enveloping Hopf algebras of Lie algebras.
- (9–10) Cell structures and derived categories in algebra and topology, triangulated categories, and maybe just a glimpse of homology and cohomology theories (K-theory, cobordism, etc).

The course web page gives links to notes on some of these topics.

The algebra sequence is under constant revision, and a constant problem is that there is too much that 'should' be covered; in particular, categorical language and homological algebra often tend to get left out of both the topology and algebra sequences, to the detriment of both.

As the last topics indicate, some algebraic topology will be combined with the algebra. The focus will be on parts of algebra, broadly understood, that are essential to ring theory, algebraic topology, algebraic geometry, and other areas of modern mathematics that require categorical language, homological algebra, and homotopical algebra.