Math 327, Spring 2011

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Course web page
http://www.math.uchicago.edu/ may/327.html

Class: MWF, 1:30 – 2:45, E206
(That is an estimated stopping time, like an airplane's estimated time of arrival.)

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In roughly their order of appearance, here are some sources. The book that comes closest to covering all that I would like to cover is Weibel's, and I have ordered copies for you (at the co-op book store).

S. Mac Lane. Categories for the working mathematician.
F. Borceux. Handbook of categorical algebra 1.
S. Mac Lane. Homology.
C.A. Weibel. An introduction to homological algebra.
H. Matsumura. Commutative ring theory.
D. Eisenbud. Commutative algebra.

I will assume that you know the things in Atiyah-MacDonald. *Nota bene: the last chapter was not covered in the Winter; some of it will be needed and may be used without giving proofs in class.* I wrote that last year and it is still true this year. That 1969 book was written from a classical point of view, "stopping short of any results requiring a deep study of homological algebra". At the time, the fundamental results relating homology to the structure theory of rings were still quite new. Of course, in one quarter we won't go very deeply either, and we will probably just barely get to derived and triangulated categories. Our main focus will be on the basics of homological algebra, with applications to commutative ring theory, finite groups, and Lie algebras and with some comparison with algebraic topology. A more detailed tentative outline is on the next page. This is a sequel to Professor Kato's Winter course, the division of material having been agreed upon between us.

Homework: An assignment will be given every week. Problems are to be turned in one week from the time they are handed out. While late problems will be accepted, they may not receive full credit. Problems are meant to help learn how to write: do not give pages of pointless detail, but do be convincing. Use your judgment.

Problems for the first five weeks are posted on the course web site:
http://www.math.uchicago.edu/ may/327.html
They may be subject to change, but with notice. I'm assuming that the problem sets do not have to be printed out for you.

There are no exams. No R's will be given to people who do most of the homework and get most of it right in a reasonably timely fashion. C's will be given if there is serious lack of understanding or if a significant portion of the homework is not done or is often done late. *Late packets of homework are unfair to the grader.* As a rough guideline, based on past experience, I expect people to complete, *accurately*, at least 75% of the homework problems. The more the better. I do look over the homeworks myself. But the problems are intended to be fun: do what you can, and don't agonize over them. Work together, but write up the problems by yourself.

This is an experimental course that mixes a range of topics focused largely on homological algebra. Topics, by week, include the following. I'm aware that this is way too ambitious, but I'm hoping you will agree that it is worthwhile to see a range of topics rather than to focus too much on any one. We may not get to the topics in the later weeks, and they are provisional.

- (1) Categorical language: limits, colimits, and universal properties; isomorphism and equivalence of categories; a bit on Abelian, monoidal, and enriched categories; maybe a tiny bit on bicategories.
- (2) Basic homological algebra: tensor products and Hom; projective and injective modules; axioms for Tor and Ext, construction and verification of the axioms; homological dimension; examples.
- (3)–(5) Review of Dedekind rings and their characterizations; number rings. Some local ring theory and dimension theory: Cohen-Macauley rings, regular local rings, their homological characterization, and the unique factorization theorem.
- (6)-(7) Some basic Lie algebra theory and a bit on restricted Lie algebras. Algebras, coalgebras, bialgebras, and Hopf algebras; the enveloping Hopf algebras of Lie algebras; maybe some structure theory and relations to algebraic geometry and topology.
- (8)-(9) An introduction to the homology of groups and its relationship to topology. An introduction to the homology of Lie algebras. Perhaps a glimpse at spectral sequences.
- (10) A piacere. Possibly a brief introduction to derived and triangulated categories in homological algebra and topology.

The course web page gives links to notes on nearly all of these topics.

The algebra sequence is under constant revision, and a constant problem is that there is way too much that 'should' be covered; in particular, categorical language, homological algebra, and the relationship between algebra and topology often tend to get left out of both the topology and algebra sequences, to the detriment of both.

As the last topics indicate, some algebraic topology may be combined with the algebra, but probably less than last year. The focus will be on parts of algebra, broadly understood, that are essential to ring theory, algebraic geometry, algebraic topology, and other areas of modern mathematics that require categorical language, homological algebra, and homotopical algebra.

This year, as an experiment, I will be giving an extemporaneous miscellany of algebraic topology beyond what is in the first year sequence. This will be in the same room, E206, in the otherwise often wasted half hour MWF 3:00 – 3:30. Attendance by first year students (or anybody else) is most certainly NOT required. It is meant to be fun for all who have any interest in algebraic topology and related areas of mathematics, such as algebraic geometry and geometric topology.