THE CLASSIFICATION OF SURFACES

Surface	Euler	orient?	genus
S	2	yes	0
nT	$2-2n$	yes	n
nP	$2-n$	no	n
K	0	no	2

$S =$ 2-sphere S^2

$T =$ torus $S^1 \times S^1$

$P =$ real projective plane $\mathbf{R}P^2$

$K =$ Klein bottle

$nC =$ connected sum of n copies of C (any C)

$C * D$ is orientable if and only if both C and D are orientable

$$\chi(C * D) = \chi(C) + \chi(D) - 2 \quad \text{(any C and D)}$$

$$S * C = C \quad \text{(any } C\text{)}$$

$$K = 2P \quad P * T = K * P \quad \Longrightarrow \quad P * T = 3P$$

$$mP * nP = (m+n)P \quad mT * nT = (m+n)T$$

$$mP * nT = (m+2n)P$$