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An Appreciation of the Work of Samuel Eilenberg (1913–1998)

I wish I had known Sammy better.[1] I remember him best from a conference at Kazimierz, Poland, in 1993. He was in fine form, but my impression was that he was in Poland then more for the memories than for the mathematics. Sammy grew up and reached mathematical maturity in Poland. He arrived in the United States on April 27, 1939, when he was 26 years old, escaping World War II. His parents, who were Jewish, stayed in Warsaw, and he exchanged letters with them until they were taken to a death camp by the Nazi occupiers of Warsaw. It is hard for the young, and in this context I include myself,[2] to even imagine what things were like when Sammy pulled up his roots.

Already in Poland, Sammy was a well established mathematician. There is a complete Bibliography of his work through 1976 in [50][3], and I was surprised to find that he had written 37 papers before leaving Poland.[4] Due to lack of time and energy, I have not attempted to read them at all carefully, so my appreciation of Sammy's work will begin with his publications after his arrival in the United States.[5] The last talk that Sammy gave in pre-war Poland was on the ideas of obstruction theory. He published a preliminary version of this in [7] and then updated it as the first paper he published after arriving in the U.S. ([8]).

[1] Reminiscences by some who knew him better than I did may be found in [1].
[2] My father was a German Jew who made it out of Germany in 1936.
[3] It also appears in [55], a translation into Polish of Mac Lane's article [54] from [50].
[4] Most were written in French and appeared in *Fundamenta Mathematicae*.
[5] That is also where *Mathematical Reviews* begins its entries of his work. A complete bibliography may be found in *Zentralblatt für Mathematik*.

Sammy was one of the most tasteful mathematicians I have met. His tastefulness included his choice of collaborators. He had four in Poland, including his teachers and thesis advisers Borsuk and Kuratowski,[6] but in view of his later career it seems surprising that all but four of his first 46 papers were solo efforts. His collaboration with Mac Lane began with his 47th paper, and from then on the vast majority of his papers were written collaboratively. Not just the work of Eilenberg and Mac Lane, Cartan and Eilenberg, and Eilenberg and Steenrod, but also the work of Eilenberg and Zilber, Eilenberg and Moore, Eilenberg and Kelly, and Eilenberg either alone or with still others transformed and reshaped vast areas of modern mathematics.

Remarkably, his sheer good taste was also powerfully expressed outside of mathematics. He was not merely a famous mathematician. He was also a famous collector of Southeast Asian art. I once went to a high end antiques store in London (intrigued by some Chinese jades) and mentioned that I was a friend of Samuel Eilenberg. From then on the owner treated me like royalty. He was quite surprised to learn that the famous Samuel Eilenberg was also a mathematician. There is an "Eilenberg collection" of over 400 items at the Metropolitan Museum of Art, donated by Eilenberg. It is a treasure trove of objects both beautiful and esoteric. For the former, let me refer you to [52]. For the latter, let me refer you to [2], the abstract for which reads:

> The stimulant known as betel, prepared from evergreen leaves and parings of the betel-nut, is enormously popular throughout much of Asia and the Pacific. It plays a major role in entertaining, in courtship and marriage, and in the traditional etiquette of the royal courts, where betel sets still form part of the state regalia. Betel cutters, the hinged implements used in the preparation of the substance, were and are items of social prestige, reflecting the wealth and taste of their owner and designed and embellished with the same care lavished on other prestige items such as jewelry and weapons. Free from any iconographic constraints, craftsmen have been able to give rein to their imagination, and the result is an enormous variety of designs ranging from classical forms in the Mughal tradition to others which can be whimsical, erotic or grotesque. This, the first full-length study devoted to betel cutters, catalogues 187 fine examples from the important collection assembled by Professor Samuel Eilenberg over almost forty years.

It seems incredible that Professor Samuel Eilenberg (as he was known in the art world) could have had the time and energy to amass such a collection, while Sammy (as he was known in the world of mathematics) was writing

[6] Sammy and Kuratowski also coauthored one of Sammy's last papers in topology [15].

seven books and over a hundred papers. It is impossible to do justice to his entire output, and I'll focus on that part of his work which I know best, mixing a chronological development with a thematic one. Thus I will consider the earliest paper on each theme, no matter how relatively inconsequential, and then briefly trace the development of that theme chronologically as it evolved to give the full flowering of the theme, very often in essentially the same form that we know it today, in all cases roughly a half century later. Since the themes were thoroughly intermingled, this cannot do justice to Sammy's work, but it is the best I can do.

1. Singular homology, simplicial sets, and acyclic models

The notion and terminology of singular simplices goes back to 1930's work of Lefschetz [51]. The term "singular" refers to maps from simplices into spaces X that lack any good behavior with respect to the structure of X. Lefschetz's work was unsatisfactory as a foundation for homology theory because its use of orientations forced there to be 2-torsion in the resulting chain complexes. Sammy's 1940's paper [8] develops obstruction theory using careful arguments with cellular cochains. There is a closely related 1940 paper that shows Sammy not to be perfect; he occasionally got things seriously wrong. In [9], he asserts that if m is odd and α_1, α_2 are any two integers, there is a mapping $\phi\colon S^m \times S^m \to S^m$ of the type [= bidegree] (α_1, α_2), which contradicts Hopf invariant one. But it must be remembered how very little algebraic topology was understood in 1940.[7]

The far more modern 1944 paper [10] is of an altogether different stature. By then, Sammy had really found his voice. In [10] he defines singular homology essentially as we know it today. His use of degeneracy operators foreshadows the introduction of what are today called simplicial sets, and the paper finally gives singular homology clear and firm foundations. In collaboration with Zilber [47], this led to the introduction of "semi-simplicial sets" which, after differing usage for many years when they were called Δ-sets are nowadays again called semi-simplicial sets. These lack degeneracy operators. They then defined "complete semi-simplicial sets" by adding in degeneracy operators, and these are the modern simplicial sets. It is curious

[7] According to Stefan Jackowski, in the Sierpiński Lecture which Sammy gave in Warsaw in 1991 he described the discovery of the Hopf map $S^3 \to S^2$, the first example of a non-null homotopic map between spheres of different dimensions, as the most crucial discovery in 20th century topology.

that the understanding that simplicial objects can be defined in any category was slow to appear, and I do not know the original source. In 1952, two years after [47] appeared, Eilenberg and Mac Lane defined "FD complexes" in [33], but in fact those are just simplicial abelian groups. A year later, in back-to-back papers, Eilenberg and Mac Lane developed the theory of acyclic models [32] and Eilenberg and Zilber proved the Eilenberg–Zilber theorem [48] establishing the equivalence between $C_*(K \times L)$ and $C_*(K) \otimes C_*(L)$ for simplicial sets K and L.

It is typical of Sammy's work to see a progression from the beginnings of a subject to its modern form as in this sequence of papers. The end point was the theory encompassing the total singular complex, singular chains, and the chain complexes associated to simplicial sets in the form still current today.

2. The development of the homology and cohomology of groups

Towards the end of World War II, Sammy and Saunders Mac Lane were working together at Columbia on trajectory analysis for antiaircraft artillery, but their celebrated collaboration had already begun in 1941. It continued through the war and long afterwards and is described by Mac Lane in [50]. All of its fruits have been collected into one 841 page volume [36].[8]

Their first joint paper appeared in 1941 [16]. That paper is almost unintelligible today and was superceded by their 1942 paper [17], which introduces the first Ext group with its now standard relationship to extensions. All groups in sight are abelian. Interestingly, the paper allows extensions of a topological group G with discrete quotient group H. It is characteristic of the mathematics of the time that the algebra and topology are not much separated. The introduction of Ext groups and their application to the universal coefficient theorem in cohomology are inextricably intertwined in this paper. I'll try to outline the gradual disentanglement of the emerging fields of algebraic topology, homological algebra, and category theory as I go along.

Higher cohomology groups are defined in [23], and one can see the modern definition in process of evolution. One can see non-Abelian group extensions in [24], a curious version of how to interpret higher cohomology groups in [25], and a particular application to Galois theory in [26]. The precise modern definition evolved from these origins.

[8] My copy is signed "To Peter, Saunders" in very large handwriting.

Analogously, a still often cited 1948 paper of Chevalley and Eilenberg [6] developed the modern foundations for the cohomology theory of Lie algebras, although again the algebra and topology are intertwined. The theory is presented so that the tangent bundles of Lie groups and the cohomology of their Lie algebras are described together in terms of a language of algebraically defined differential forms.

3. The introduction of category theory

Going back to 1942, we find the first announcement [18] of what was to become category theory. There Eilenberg and Mac Lane only discussed manifestations in group theory of the notion of naturality that they saw all around them. The fully general language of category theory as we still know it today was introduced in the hugely influential paper [19]. There surely cannot ever have been a single paper with not a single real theorem that has had a more significant role in the development of mathematics. A great deal of modern mathematics would quite literally be unthinkable without the language of categories, functors, and natural transformations introduced by Eilenberg and Mac Lane in 1945. It was perhaps inevitable that some such language would have appeared eventually. It was certainly not inevitable that such an early systematization would have proven so remarkably durable and appropriate; it is hard to imagine that this language will ever be supplanted.

4. The foundations of algebraic topology

The language of category theory was essential to the formulation of the Eilenberg–Steenrod axioms for homology and cohomology theories. The axiomatization was announced in 1945 [45], but the authorative book on the subject did not appear until 1952 [46]. Even then, it was only half of the project. As Cartan's review makes clear [4], [46] was meant to be the first of two volumes, and Cartan was informed of the contents of the sequel:

> Le volume 2 doit contenir des développements sur le calcul pratique de l'homologie d'un espace muni d'une décomposition cellulaire, six chapitres sur la théorie des produits (cross, cup et cap product), et une axiomatisation de l'homologie à coefficients locaux.

It is a pity the second volume never appeared. The cited products are still poorly documented in the literature,[9] and I still do not know of an axiomatization of homology with local coefficients.

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[9] I tried to clarify things homotopically in [58, §6].

The clear separation of the homology of chain complexes from the homotopy theoretical nature of the axiomatization foreshadows the subsequent development of generalized cohomology theories. As has been frequently remarked, it is a nice fact that the dimension axiom came last and that it can well be dropped in much of the formal development. The separation is essential, since a generalized cohomology theory computable by chain complexes must be a product of ordinary cohomology theories. This book is so very well-known that I will say no more about it. Younger readers are encouraged to read it. They might want to replace the word polyhedron throughout by CW complex.

5. Relations between homology and homotopy groups

In retrospect, an astonishing omission from Eilenberg and Steenrod [46] is the lack of any mention of Eilenberg–Mac Lane spaces. Perhaps that is accounted for by the lack then of a single space axiomatization of (reduced) cohomology. The use of relative cohomology obscures the role of Eilenberg–Mac Lane spaces. (See [57] for some relevant history). In any case, the first mention of Eilenberg – Mac Lane spaces is in an announcement from 1943 [20] of results to come concerning the relationship between homology and homotopy groups of spaces. A related announcement from 1946 [22] gives the first mention of a k-invariant, namely k^3. A long and detailed sequence of joint papers develops these ideas in detail. These papers make for hard but still interesting reading nowadays. In [21], for example, it is explained how to compute the cohomology groups of a space X whose only nonvanishing homotopy group is $\pi_1(X)$ in terms of a space $K(\pi_1(X))$. To a modern reader, X is itself a space $K(\pi_1(X), 1)$, but Eilenberg and Mac Lane have in mind a very specific combinatorial construction of a space $K(\pi_1(X), 1)$.

Sammy alone wrote a beautiful 1947 paper [11] that, among other things, explains how to simultaneously generalize group cohomology and the cohomology of spaces with local coefficients. In particular, it shows how to see the group cohomology of modules with non-trivial group action topologically.[10] It was used in a 1947 sequel by Eilenberg and Mac Lane [27], which shows how to compute $H^q(X; A)$ for a space X such that $\pi_i(X) = 0$ for $1 < i < q$ in terms of $\pi_1(X)$, the $\pi_1(X)$-group $\pi_q(X)$ and the k-invariant k^{q+1}. The case $q = 2$ had been announced in [22]. The methods are primarily homotopi-

[10] A student of mine, Megan Shulman, showed in her 2010 thesis that it adapts directly to modern work in equivariant cohomology theory.

cal. Their closely related 1950 paper [28] studies such spaces X using the methods of singular homology that had been developed in the interim. One reason these papers make for hard reading today, and one reason Eilenberg and Mac Lane did not themselves introduce Postnikov towers, is that they never make simplifying assumptions about $\pi_1(X)$ and its action on the higher homotopy groups $\pi_n(X)$. Their level of generality created substantial difficulties that disappear upon restriction to simple spaces.

6. Eilenberg–Mac Lane spaces

The announcements [29–31] and papers [33–35] are devoted to the study of what are now called Eilenberg–Mac Lane spaces. In retrospect, these papers are far more important for the methods they introduced than for the actual calculations, which were soon to be superceded by those of Cartan [5] and which for the most important cases of groups $\mathbb{Z}/p\mathbb{Z}$ of integers mod p were reworked definitively in terms of Steenrod operations by Serre [62] at $p = 2$ and Cartan, also in [5], at $p > 2$. For example, the bar construction introduced by Eilenberg and Mac Lane is now a ubiquitous tool with many variants and applications.

One reason for the modern difficulty with reading these papers is that Eilenberg and Mac Lane tried to work with a general abelian group π and obtain the cohomology groups of $K(\pi, n)$ as a functor of π. This is a theme that Mac Lane and his students were still pursuing decades later, and there is still no definitive answer. A related difficulty is that they worked with several functorial models for the spaces and simplicial sets $K(\pi, n)$ and for their associated chain complexes. The functoriality forced these complexes to be very large. Cartan saw how to inductively construct much smaller and more computable approximations to these chain complexes, not directly related to the topology of the spaces $K(\pi, n)$.

7. The foundations of homological algebra

We now come to the magisterial 1956 book [3] of Cartan and Eilenberg. I love this book, and I am not the only one. It is by far the most cited of Eilenberg's works. In this book they establish homological algebra as a new branch of mathemematics. The democratic emphasis that, as a matter of both theory and calculation, any projective or injective resolutions can be used now seems entirely obvious, but to the best of my knowledge it is new to this book. While there were of course many precursors, the definitions of Tor and Ext in their natural level of generality for rings and modules first appear here. While some of the theory, especially the theory of satellites, is obsolete, the theory of derived functors is already well developed, and again, this is where it first appears. What is especially appealing is the mix of theory with examples and calculations. Of course, modern books contain new material developed in the more than half century since [3] was written, but [3] is still at least as readable as any modern source for the fundamentally important topics it covers. All subsequent treatments of homological algebra start from the foundations Cartan and Eilenberg gave us.

8. The Eilenberg–Moore spectral sequence and related work

My thesis advisor John Moore was another collaborator of Eilenberg. They introduced what is now called the Eilenberg–Moore spectral sequence [39]. As a preliminary, they first published a helpful paper concerning the convergence of spectral sequences in general [38]. To make a point that illustrates Sammy's sheer tastefulness, I will go into just a little detail about the mathematics here. Suppose given a fibration $p\colon E \to B$ and a map $f\colon A \to B$ and let $q\colon D \to A$ be the pullback fibration. Under mild hypotheses, and assuming for simplicity that we are working over a field of coefficients, the Eilenberg–Moore spectral sequence converges from $E_2 = \mathrm{Tor}^{H^*(B)}(H^*(A), H^*(E))$ to $H^*(D)$. When working more generally over a commutative ring rather than a field, there is a question of whether the E_2 term should be a relative Tor or an absolute Tor.[11] Perhaps this is what led Eilenberg and Moore to their valuable memoir [40] that establishes the foundations of relative homological algebra.

The Eilenberg–Moore spectral sequence is a kind of dual to a spectral sequence implicit in the work of Eilenberg and Mac Lane that was later made explicit by Milnor [60] and Rothenberg and Steenrod [61]. That spectral sequence converges from $E^2 = \mathrm{Tor}^{H_*(G)}(H_*(X), H_*(Y))$ to $H_*(B(Y, G, X))$, where G is a topological group that acts from the right on Y and from the left on X and $B(Y, G, X)$ is the two-sided bar construction. The latter spectral sequence works just as well with H_* replaced by a suitable generalized homology theory, but the former is special to ordinary cohomology, the basic point being that the two-sided geometric cobar construction $C(A, B, E)$ is not equivalent to D in the context of the Eilenberg–Moore spectral sequence.

[11] In fact, with different constructions, either is possible [49].

Sammy was not satisfied with the duality just described. It seemed distasteful, and there were awkward finite type hypotheses in the development of the Eilenberg–Moore spectral sequence. The 1966 paper [19] of Eilenberg and Moore gave a more tasteful description, working in homology and replacing $\mathrm{Tor}^{H^*(B)}(H^*(A), H^*(E))$ with the genuinely dual $\mathrm{Cotor}_{H_*(B)}(H_*(A), H_*(E))$, where Cotor is defined for comodules over a coalgebra. This may still not be as widely used as it ought to be.

In a peripherally related 1965 paper in category theory [18], Eilenberg and Moore introduced the by now standard Eilenberg–Moore adjunction between a category and the category of algebras over a monad on that category. In one of Sammy's few lapses of taste, the silly word triple, which by now I hope is obsolete, was used instead of monad, but that was before I persuaded Mac Lane to switch from triple to monad when writing his canonical introduction to category theory [56].[12]

9. A few other contributions

9.1. *Quaternionic fundamental theorem of algebra.* I can't resist mentioning a lovely 1944 note [44] (with Niven) that gives a homotopical proof of a natural version of the fundamental theorem of algebra for the quaternions rather than for the complex numbers.

9.2. *Fixed point theory.* I do not need to emphasize to a Polish audience how important and vibrant fixed point theory remains today. This was one of Sammy's continued interests. His 1946 paper [37] (with Deane Montgomery) is still frequently cited, even though its language of multi-valued transformations seems to be little known by modern algebraic topologists (at least in the U.S.).

9.3. *Homological dimension theory.* In the 1950's, there was a long series of papers by a number of authors in which the methods of homological algebra were systematically applied to the structure theory of rings and algebras. Sammy was an author or coauthor of quite a few of these papers, and his influence on this fundamentally important part of modern algebra is apparent. I especially like [43], with Nakayama, in which the modern understanding of quasi-Frobenius rings and Frobenius algebras in terms of homological dimension was first explained.

[12] I tell the story in [59].

9.4. *Closed categories.* I am very fond of the 1966 paper [14], with Max Kelly. Coherence theorems in category theory assert intuitively that all diagrams that should commute do commute. Saunders Mac Lane [53] introduced the precise definition of symmetric monoidal categories and proved the basic coherence theorem for them, which shows that the short list of diagrams included in his definition implies that all diagrams that can be expected to commute do in fact commute. Adding a closed structure means adding an internal hom functor satisfying the expected adjunction $\mathrm{Hom}(A \otimes B, C) \cong \mathrm{Hom}(A, \mathrm{Hom}(B, C))$. This paper formalizes what diagrams involving Hom can be expected to commute, and it proves that the adjunction itself already implies that they do commute: no further coherence axioms are required. This is not at all obvious.

10. Automata theory

I am not sufficiently knowledgable to begin to do justice to Eilenberg's new direction towards the end of his mathematical activity. The two volumes of his authorative book on automata theory [12, 13] appeared in 1974 (the year Sammy turned 60) and 1976, and they are still standard references in that field today. To quote from *Mathematical Reviews* [63],

> The publication of this multi-volume treatise is, in the reviewer's opinion, one of the most important events in the mathematical study of the foundations of computer science and in applied mathematics. The work includes a unifying mathematical presentation of almost all major topics of automata and formal language theory.

It seems that establishing the modern unifying framework of algebraic topology, homological algebra, and category theory by clearly exposing the foundations of these subjects was not enough for Sammy. He ended his major work by also establishing the modern unifying framework of automata theory. It is given to very few to have such a profound and long-lasting influence on mathematics.

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