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STABLE MAPS BETWEEN CLASSIFYING SPACES

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The purpose of this short note is to advertise and reprove the following remarkable result of Goro Nishida [8]. Let G and Π be finite groups with p-Sylow subgroups G_p and Π_p.

Theorem 1. If BG is p-locally a stable retract of $B\Pi$, then G_p is isomorphic to a subgroup of Π_p.

The hypothesis means that the localization at p of the suspension spectrum of BG is a retract of that of $B\Pi$. If these localizations are equivalent, we can conclude more.

Corollary 2. If BG and $B\Pi$ are p-locally stably equivalent, then G_p and Π_p are isomorphic.

This is surprising because BG is p-locally a stable retract and thus a stable wedge summand of BG_p. The corollary implies that data about this wedge summand is forcing information about the complementary wedge summand.

Since a nilpotent group is the product of its p-Sylow subgroups, we have the following immediate consequence.

Corollary 3. If G and Π are nilpotent and BG and $B\Pi$ are stably equivalent, then G and Π are isomorphic.

It can be shown by example that stable equivalence fails to imply isomorphism for general finite groups. However, the following weaker conclusion is well-known.

Corollary 4. If $\phi: G \to \Pi$ is a homomorphism which induces an isomorphism on integral homology, then ϕ is an isomorphism.

Proof. Since $B\phi$ is a homology isomorphism, it is a stable equivalence. Let $\Lambda = \mathrm{Im}(\phi)$. Since $B\phi$ factors through $B\Lambda$, BG is a stable retract of $B\Lambda$.

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By the theorem and the fact that Λ is a quotient of G, the groups G_p, Λ_p, and Π_p are isomorphic for each prime p. Thus G, Λ, and Π all have the same order. Since G and Λ have the same order, ϕ is a monomorphism; since G and Π have the same order, ϕ is an isomorphism.

In fact, as we shall explain, Theorem 1 is a direct application of an earlier result of Lewis, McClure, and myself [3] which gives a complete algebraic description of the group of stable maps $BG \to B\Pi$. In turn, we were led to this description by ideas and questions of Adams, Gunawardena, and Miller (see [1, §9]).

We begin with the relevant algebraic definitions. Let $A^+(G,\Pi)$ be the semi-group of isomorphism classes of finite Π-free $(G \times \Pi)$-sets, addition being disjoint union. We let G act from the left and Π from the right on such sets. Let $A(G,\Pi)$ be the associated Grothendieck group. Clearly $A(G,\Pi)$ is the free Abelian group generated by the transitive $(G \times \Pi)$-sets in $A^+(G,\Pi)$. Each such set S has the form $(G \times \Pi)/\Delta\rho$, where ρ is a homomorphism from some subgroup H of G to Π and

$$\Delta\rho = \{(h,\rho(h)) \mid h \in H\}.$$

Two such S are isomorphic if and only if the corresponding subgroups of $G \times \Pi$ are conjugate. We prefer to express $(G\times\Pi)/\Delta\rho$ in the equivalent form

$$G \times_\rho \Pi = (G \times \Pi)/(\sim), \quad \text{where } (gh,\pi) \sim (g,\rho(h)\pi)$$

for $g \in G$, $h \in H$, and $\pi \in \Pi$. The left action of G and right action of Π are evident.

We shall later need certain subgroups of $A(G,\Pi)$. Let $A'(G,\Pi)$ be the Grothendieck group of those $S \in A^+(G,\Pi)$ such that S/Π is G-fixed point free. Clearly $A'(G,\Pi)$ is spanned by those $G \times_\rho \Pi$ such that ρ is defined on a proper subgroup H of G. Let $A_s(G,\Pi)$ be the Grothendieck group of G-singular sets $S \in A^+(G,\Pi)$; that is, we require each $s \in S$ to be fixed by some $g \neq e$ in G. It is easy to check that $A_s(G,\Pi)$ is spanned by those $G \times_\rho \Pi$ such that ρ is not a monomorphism; when ρ is a monomorphism, $G \times_\rho \Pi$ is G-free. Thus the quotient

$$A(G,\Pi)/(A'(G,\Pi) + A_s(G,\Pi))$$

is free Abelian on those $G \times_\rho \Pi$ (if any!) such that ρ is a monomorphism defined on all of G. The relevance to Theorem 1 is clear.

We shall also need various maps relating the groups $A(G,\Pi)$ as G and Π vary. These will model corresponding operations between stable maps, and we shall give more information than is relevant to Theorem 1. For a third group Γ, we have a composition pairing

$$\gamma: A(\Pi,\Gamma) \otimes A(G,\Pi) \to A(G,\Gamma).$$

It is specified on $T \in A^+(\Pi,\Gamma)$ and $S \in A^+(G,\Pi)$ by

$$\gamma(T \otimes S) = S \times_\Pi T = (S \times T)/(\sim), \text{ where } (s\pi,t) \sim (s,\pi t)$$

for $s \in S$, $\pi \in \Pi$, and $t \in T$. It is easy to check the following closure properties.

Lemma 5. The pairing γ restricts to pairings

$$A(\Pi,\Gamma) \otimes A'(G,\Pi) \to A'(G,\Gamma)$$

and

$$A(\Pi,\Gamma) \otimes A_s(G,\Pi) \to A_s(G,\Gamma).$$

It is also easy to give an explicit formula for γ (although we shall not have occasion to use it).

Lemma 6. Let $S = G \times_\rho \Pi$, $\rho: H \to \Pi$, and $T = \Pi \times_\sigma \Gamma$, $\sigma: \Lambda \to \Gamma$. Let $K = \mathrm{Im}(\rho)$ and let $\{\pi\} \subset \Pi$ be a set of double coset representatives for $K\backslash\Pi/\Lambda$. Let $\Lambda^\pi = \pi\Lambda\pi^{-1}$ and let $c(\pi):\Lambda^\pi \to \Lambda$ be the conjugation isomorphism. Define ξ_π to be the composite

$$\rho^{-1}(K \cap \Lambda^\pi) \xrightarrow{\rho} K \cap \Lambda^\pi \subset \Lambda^\pi \xrightarrow{c(\pi)} \Lambda \xrightarrow{\sigma} \Gamma.$$

Then $S \times_\Pi T$ is isomorphic as a $(G \times \Gamma)$-set to $\coprod_\pi G \times_{\xi_\pi} \Gamma$.

Proof. Define $\phi_\pi: G \times_{\xi_\pi} \Gamma \to S \times_\Pi T$ by $\phi_\pi(g,\gamma) = ((g,e),(\pi,\gamma))$ for $g \in G$ and $\gamma \in \Gamma$. Then ϕ_π is a well-defined injection of $(G \times \Gamma)$-sets, and $S \times_\Pi T$ is the disjoint union of the images of the ϕ_π.

For a monomorphism $\psi:\Lambda \to \Pi$ and any homomorphism $\phi:H \to G$, we have a restriction homomorphism

$$\beta: A(G,\Pi) \to A(H,\Lambda).$$

It is specified by $\beta(S) = {}_\phi S_\psi$, where ${}_\phi S_\psi$ denotes S regarded as a left H and right Λ set by pullback along ϕ and ψ. When ϕ and ψ are inclusions, we use the notation ${}_H S_\Lambda$ (and delete H if $H = G$ or Λ if $\Lambda = \Pi$). We may express restriction in terms of composition since

(1) $$_\phi S_\psi = {}_\phi G \times_G S \times_\Pi \Pi_\psi = \gamma(\Pi_\psi \otimes S \otimes_\phi G).$$

Let $A(G) = A(G,e)$, where e is the trivial group; this is just the usual Burnside ring of G. We have homomorphisms

$$\varepsilon: A(G,\Pi) \to A(G) \quad \text{and} \quad \eta: A(G) \to A(G,\Pi)$$

specified by $\varepsilon(S) = S/\Pi$ for $S \in A^+(G,\Pi)$ and $\eta(R) = R \times \Pi$ for $R \in A^+(G)$. These too can be expressed in terms of composition since

(2) $$\varepsilon(S) = S \times_\Pi 1 = \gamma(1 \otimes S) \quad \text{and} \quad \eta(R) = R \times \Pi = \gamma(\Pi \otimes R),$$

where $1 \in A^+(\Pi)$ is the trivial left Π-set with one element and $\Pi \in A^+(e,\Pi)$ is Π

regarded as a free right Π-set. Note that $A(e,\Pi)$ is a copy of Z with Π as generator.

Obviously $\varepsilon\eta$ is the identity map of $A(G)$. Define $K(G,\Pi) = \mathrm{Ker}(\varepsilon)$. We then have a direct sum decomposition

$$A(G,\Pi) = K(G,\Pi) \oplus \eta A(G).$$

For $Y \in A(G,\Pi)$, let $\tilde{Y}$ denote the component of Y in $K(G,\Pi)$. Thus $\tilde{Y} = Y - \eta\varepsilon(Y)$ and, in particular, $\tilde{S} = S - ((S/\Pi) \times \Pi)$. We need a smidgen of explicit calculation.

<u>Lemma 7</u>. Let H be a non-trivial subgroup of G and consider the $(H{\times}G)$-set ${}_HG$ and the $(G \times H)$-set G_H. In $A(H,H)$,

$$\gamma(\tilde{G}_H \otimes {}_H\tilde{G}) \equiv \Sigma\, {}_{c(g)}H \mod A'(H,H) + A_s(H,H),$$

where g runs through a set of coset representatives for $WH = NH/H$ and $c(g){:}H \to H$ is conjugation, $c(g)(h) = g^{-1}hg$.

<u>Proof</u>. Let $t{:}H \to G$ be the trivial homomorphism, $t(h) = e$. We have $\tilde{G}_H = G_H - (G/H) \times H$ and, since ${}_HG/G = 1$, ${}_H\tilde{G} = {}_HG - {}_tG$. Thus

$$\begin{aligned}\gamma(\tilde{G}_H \otimes {}_H\tilde{G}) &= {}_HG \times_G G_H - {}_tG \times_G G_H - {}_HG \times_G (G/H \times H) + {}_tG \times_G (G/H \times H)\\ &= {}_HG_H - {}_tG_H - {}_H(G/H) \times H + {}_t(G/H) \times H\\ &= {}_HG_H - {}_H(G/H) \times H,\end{aligned}$$

the last equality holding since ${}_tG_H$ and ${}_t(G/H) \times H$ are isomorphic $(H \times H)$-sets. (Scholium: [8, p.18] gives ${}_HG_H - {}_tG_H$ as the answer here.) Since $H \neq e$, it is clear that ${}_H(G/H) \times H$ is in $A_s(H,H)$. Let $\{g\} \subset G$ be a set of double coset representatives for $H\backslash G/H$, so that ${}_HG_H = \coprod_g HgH$ as an $(H \times H)$-set. Clearly $(HgH)/H = 1$ if and only if $hg \in gH$ for all $h \in H$, that is, if and only if $g \in NH$. Thus, for $g \notin NH$, HgH is in $A'(H,H)$. For $g \in NH$, $h \to gh$ specifies an $(H \times H)$-isomorphism ${}_{c(g)}H \to HgH$, and of course we have one such double coset representative g in each coset of NH/H. (Scholium: [8, p.18] says these HgH are all $(H{\times}H)$-isomorphic.)

For a second pair of finite groups (G',Π'), we have the pairing

$$\wedge{:}A(G,\Pi) \otimes A(G',\Pi') \to A(G \times G', \Pi \times \Pi')$$

given by Cartesian products of finite sets. In particular, using the restriction associated to the diagonal $G \to G \times G$, we see that $A(G,\Pi)$ is an $A(G)$-module. Of course, there is a whole slew of formal identities and coherence isomorphisms relating the various operations we have introduced.

Turning to spaces, we agree to use the same letter to denote a based space and its suspension spectrum, and similarly for maps. We let X_+ denote the union of a space X and a disjoint basepoint. A set $S \in A^+(G,\Pi)$ may be viewed as the total space of a principal Π-bundle with G action through bundle maps. We let EG be the standard contractible free G-space and have the principal Π-bundle

$$\xi(S):EG \times_G S \to EG \times_G S/\Pi.$$

We also write $\xi(S)$, or ξ for short, for its classifying map

$$(EG \times_G S/\Pi)_+ \to B\Pi_+ .$$

We have a (not necessarily connected) finite cover

$$EG \times_G S/\Pi \to EG \times_G 1 = BG,$$

and we write $\tau(S)$, or τ, for its stable transfer map

$$BG_+ \to (EG \times_G S/\Pi)_+ .$$

Both ξ and τ are additive in S, and we define

$$\alpha:A(G,\Pi) \to [BG_+,B\Pi_+]$$

to be the unique homomorphism such that $\alpha(S) = \xi(S) \circ \tau(S)$, where [X,Y] denotes the Abelian group of stable maps $X \to Y$. For $H \subset G$, $EG \times_G G/H = EG/H$ is a model for BH, and we write ι or ι_H^G for the natural cover $BH \to BG$ and τ or τ_H^G for its transfer. If $S = G \times_\rho \Pi$, $\rho:H \to \Pi$, then $S/\Pi = G/H$ and $\alpha(S)$ is the composite

$$BG_+ \xrightarrow{\tau_H^G} BH_+ \xrightarrow{B\rho} B\Pi_+ .$$

In particular, if $H = G$, then $S = {}_\rho\Pi$ and $\alpha(S) = B\rho$. Clearly $\alpha({}_HG) = \iota_H^G$ and $\alpha(G_H) = \tau_H^G$; this explains the interest of Lemma 7.

We relate γ, β, ε, η, and $\wedge$ to operations between stable maps. We could use Lemma 6 to prove the following result, but we prefer to give a conceptual argument due to Adams.

Proposition 8. The following diagram commutes.

$$\begin{array}{ccc} A(\Pi,\Gamma) \otimes A(G,\Pi) & \xrightarrow{\gamma} & A(G,\Gamma) \\ \downarrow{\scriptstyle \alpha \otimes \alpha} & & \downarrow{\scriptstyle \alpha} \\ [B\Pi_+,B\Gamma_+] \otimes [BG_+,B\Pi_+] & \xrightarrow{\text{composition}} & [BG_+,B\Gamma_+] \end{array}$$

Proof. Let $S \in A^+(G,\Pi)$ and $T \in A^+(\Pi,\Gamma)$ and observe that

$$(S \times_\Pi T)/\Gamma = S \times_\Pi (T/\Gamma).$$

We have a Π-bundle map $\tilde{\xi}(S):EG \times_G S \to E\Pi$ with base map $\xi(S)$ and a Γ-bundle map $\tilde{\xi}(T):E\Pi \times_\Pi T \to E\Gamma$ with base map $\xi(T)$. The top two squares are maps of principal Γ-bundles and the bottom square is a pullback in the diagram

$$\begin{array}{ccccc}
EG \times_G S \times_\Pi T & \xrightarrow{\tilde{\xi}(S) \times 1} & E\Pi \times_\Pi T & \xrightarrow{\tilde{\xi}(T)} & E\Gamma \\
\downarrow & & \downarrow & & \downarrow \\
EG \times_G S \times_\Pi T/\Gamma & \xrightarrow{\tilde{\xi}(S) \times 1} & E\Pi \times_\Pi T/\Gamma & \xrightarrow{\xi(T)} & B\Gamma \\
\downarrow & & \downarrow & & \\
EG \times_G S/\Pi & \xrightarrow{\xi(S)} & B\Pi & &
\end{array}$$

By the naturality of transfer on pullbacks, there results a commutative diagram

$$\begin{array}{ccccc}
BG_+ & \xrightarrow{\tau(S)} & (EG \times_G S/\Pi)_+ & \xrightarrow{\tau} & (EG \times_G S \times_\Pi T/\Gamma)_+ \\
& \searrow^{\alpha(S)} & \downarrow{\xi(S)} & & \downarrow{\tilde{\xi}(S) \times 1} \\
& & B\Pi_+ & \xrightarrow{\tau(T)} & (E\Pi \times_\Pi T/\Gamma)_+ \\
& & & \searrow^{\alpha(T)} & \downarrow{\xi(T)} \\
& & & & B\Gamma_+
\end{array}$$

By the transitivity of transfer, the horizontal composite is $\tau(S \times_\Pi T)$. By the first diagram, the vertical composite is $\xi(S \times_\Pi T)$. Thus $\alpha(T)\alpha(S) = \alpha(S \times_\Pi T)$.

<u>Corollary 9</u>. The following diagram commutes for a monomorphism $\psi:\Lambda \to \Pi$ and a homomorphism $\phi:H \to G$.

$$\begin{array}{ccc}
A(G,\Pi) & \xrightarrow{\beta} & A(H,\Lambda) \\
\downarrow{\alpha} & & \downarrow{\alpha} \\
[BG_+, B\Pi_+] & \xrightarrow{[B\phi,\tau(\psi)]} & [BH_+, B\Lambda_+]
\end{array}$$

<u>Proof</u>. $B\phi = \alpha({}_\phi G)$, while $\tau(\psi) = \alpha(\Pi_\psi)$ is the transfer associated to $B\psi:B\Lambda \to B\Pi$. The conclusion is immediate from formula (1).

Let $\varepsilon:B\Pi_+ \to S^0$ send all of $B\Pi$ to the non-basepoint of S^0 and let $\eta:S^0 \to B\Pi_+$ send the non-basepoint to the basepoint in $B\Pi$. In fact, Be is a point, hence $Be_+ = S^0$, and ε and η are induced by the trivial homomorphism $\Pi \to e$ and the inclusion $e \to \Pi$.

<u>Corollary 10</u>. The following diagram commutes.

$$\begin{array}{ccccc}
A(G) & \xrightarrow{\eta} & A(G,\Pi) & \xrightarrow{\varepsilon} & A(G) \\
\downarrow{\alpha} & & \downarrow{\alpha} & & \downarrow{\alpha} \\
[BG_+, S^0] & \xrightarrow{\eta_*} & [BG_+, B\Pi_+] & \xrightarrow{\varepsilon_*} & [BG_+, S^0]
\end{array}$$

Proof. Clearly $\alpha(1) = \varepsilon$ and $\alpha(\Pi) = \eta$, where $1 \in A(\Pi)$ and $\Pi \in A(\Pi, e)$ are as in formula (2). The conclusion follows.

Of course, $\varepsilon\eta : S^0 \to S^0$ is the identity and $B\Pi_+$ splits stably as the wedge of $B\Pi$ and S^0, and similarly for G. Since $[S^0, B\Pi] = 0$, the kernel of ε_* may be identified with $[BG, B\Pi]$.

Corollary 11. $\alpha : A(G,\Pi) \to [BG_+, B\Pi_+]$ restricts to a homomorphism

$$K(G,\Pi) \to [BG, B\Pi].$$

Proposition 12. The following diagram commutes.

$$\begin{array}{ccc} A(G,\Pi) \otimes A(G',\Pi') & \xrightarrow{\wedge} & A(G \times G', \Pi \times \Pi') \\ \downarrow{\scriptstyle \alpha \otimes \alpha} & & \downarrow{\scriptstyle \alpha} \\ [BG_+, B\Pi_+] \otimes [BG'_+, B\Pi'_+] & \xrightarrow{\wedge} & [B(G \times G')_+, B(\Pi \times \Pi')_+] \end{array}$$

Proof. The suspension spectrum functor commutes with smash products, $X_+ \wedge Y_+ = (X \times Y)_+$ on the space level, and the classifying space functor commutes with products. Under the resulting identifications, the transfer commutes with products. (See [4] for a proof of this and of all other facts used about transfer.) Together with an obvious inspection of $(\Pi \times \Pi')$-bundle maps, this shows that, for $S \in A^+(G,\Pi)$ and $S' \in A^+(G',\Pi')$, the composite

$$BG_+ \wedge BG'_+ \xrightarrow{\tau(S) \wedge \tau(S')} (EG \times_G S/\Pi)_+ \wedge (EG' \times_{G'} S'/\Pi')_+ \xrightarrow{\xi(S) \wedge \xi(S')} B\Pi_+ \wedge B\Pi'_+$$

may be identified with the composite

$$B(G \times G')_+ \xrightarrow{\tau(S \times S')} (E(G \times G') \times_{G \times G'} (S \times S')/(\Pi \times \Pi'))_+ \xrightarrow{\xi(S \times S')} B(\Pi \times \Pi')_+ .$$

Giving $[BG_+, S^0]$ an A(G)-module structure by pullback along the ring homomorphism $\alpha : A(G) \to [BG_+, S^0]$, we see that the evident $[BG_+, S^0]$-module structure gives $[BG_+, B\Pi_+]$ an A(G)-module structure consistent under α with the A(G)-module structure on $A(G,\Pi)$. All maps appearing in Corollaries 10 and 11 are morphisms of A(G)-modules. Let IG be the augmentation ideal of A(G) and let $\hat{M}$ denote the completion of an A(G)-module M with respect to the IG-adic topology, $\hat{M} = \lim M/(IG)^n M$. With these notations, Lewis, McClure, and I prove the following result [3].

Theorem 13. For any finite groups G and Π, α extends to an isomorphism

$$\hat{\alpha}:\hat{A}(G,\Pi) \to [BG_+,B\Pi_+].$$

Therefore $\hat{\alpha}$ restricts to an isomorphism

$$\hat{K}(G,\Pi) \to [BG,B\Pi].$$

In fact, if $W\rho = N\Delta\rho/\Delta\rho$ for $\Delta\rho = \{(h,\rho(h))\} \subset G \times \Pi$ (as above), we construct an explicit map of spectra

$$\bigvee_\rho BW\rho_+ \to F(BG_+,B\Pi_+)$$

such that the function spectrum on the right is a suitable completion of the wedge sum on the left; here we take one ρ for each basis element $G \times_\rho \Pi$ of $A(G,\Pi)$. The theorem stated results on passage to π_0. When $\Pi = e$, the theorem is the original form of the Segal conjecture. The idea of the generalization runs as follows. An induction theorem of McClure and myself [6] reduces the problem to the case when G is a p-group. A transfer argument then reduces the problem to the case when Π is also a p-group. Here a diagram relating the maps α of the Segal conjecture for Π and $G \times \Pi$ to the maps α for the pairs $(G,W\Lambda)$, $\Lambda \subset \Pi$, allows an inductive reduction to the Segal conjecture for p-groups. The latter had not yet been proven when [3] and [6] were written, but has since been established by work of Carlsson and others [2,1,7]; see [5] for an overview. The following special case of [6, Lemma 5] gives a preliminary reduction.

Lemma 14. If G is a p-group, then the IG-adic topology coincides with the p-adic topology on the kernel of the restriction $A(G,\Pi) \to A(e,\Pi) \cong Z$.

This implies the following specialization of Theorem 13.

Corollary 15. If G is a p-group, then $\alpha:A(G,\Pi) \to [BG_+,B\Pi_+]$ is a monomorphism and α induces isomorphisms

$$A(G,\Pi)^\wedge_p \to [BG_+,B\Pi_+]^\wedge_p \quad \text{and} \quad K(G,\Pi)^\wedge_p \to [BG,B\Pi]$$

upon p-adic completion.

Here $[BG,B\Pi_+]$ is already complete at p, so p-adic completion of $[BG_+,B\Pi_+]$ only entails p-adic completion of the summand $Z = [S^0,S^0]$.

We can now prove Theorem 1. Let $f:BG \to B\Pi$ and $k:B\Pi \to BG$ be stable maps such that, p-locally, $kf \simeq 1$. Of course, the suspension spectrum of BG is trivial away from the order of G, hence it splits as the wedge of its localizations at the primes dividing the order of G. Thus k and f need only be given p-locally. Let $\tilde{h}$ denote the summand $BG \to B\Pi$ of a map $h:BG_+ \to B\Pi_+$; that is, $\tilde{h} = h - \eta_*\varepsilon_*(h)$. It is standard that the composite

$$B\Pi \xrightarrow{\tilde{\tau}} B\Pi_p \xrightarrow{\tilde{\iota}} B\Pi$$

is a p-local stable equivalence. Let ζ be a p-local stable inverse to $\tilde{\iota}\,\tilde{\tau}$ and define f_p and k_p to be the respective composites

$$BG_p \xrightarrow{\tilde{\iota}} BG \xrightarrow{f} B\Pi \xrightarrow{\tilde{\tau}} B\Pi_p$$

and

$$B\Pi_p \xrightarrow{\tilde{\iota}} B\Pi \xrightarrow{\zeta} B\Pi \xrightarrow{k} BG \xrightarrow{\tilde{\tau}} BG_p .$$

Obviously $k_p f_p$ is homotopic to the composite

$$BG_p \xrightarrow{\tilde{\iota}} BG \xrightarrow{\tilde{\tau}} BG_p .$$

By Corollary 15, there is an element $X \in K(G_p,\Pi_p)$ such that $\alpha(X) \equiv f_p$ mod p and an element $Y \in K(\Pi_p,G_p)$ such that $\alpha(Y) \equiv k_p$ mod p. By Proposition 8, we have

$$\alpha\gamma(Y \otimes X) = \alpha(Y)\alpha(X) \equiv k_p f_p = \tilde{\tau}\,\tilde{\iota} \text{ mod p.}$$

Here $\tilde{\tau} = \alpha(\tilde{G}_H)$ and $\tilde{\iota} = \alpha({}_H\tilde{G})$, $H = G_p$. We may assume that $G_p \neq e$ (since Theorem 1 is trivial if $G_p = e$), and we conclude from Lemma 7 and the injectivity of α that

$$\gamma(Y \otimes X) \equiv \Sigma_{c(g)}G_p \text{ mod (p)} + A'(G_p,G_p) + A_s(G_p,G_p),$$

where g runs through a set of coset representatives for WG_p. Since the order of WG_p is prime to p, $\gamma(Y \otimes X) \not\equiv 0$. Therefore, by Lemma 5,

$$X \not\equiv 0 \text{ mod } A'(G_p,\Pi_p) + A_s(G_p,\Pi_p)$$

and $A(G_p,\Pi_p)$ must have at least one basis element $G_p \times_\rho \Pi_p$ such that ρ is a monomorphism defined on all of G_p.

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