MEMORIAL TRIBUTE: MARK STEINBERGER (1950–2018)

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Mark Steinberger died of brain cancer on September 15, 2018. He was both a graduate student and a relative of mine. I knew him as a child, although not well. His father's mother was my father's sister. That side of our family escaped from Nazi Germany in the 1930's. As teenagers, Mark's father Herbert and his brother Jack were sent to the United States on the first Kindertransport out of Germany in 1934. Their parents and younger brother Rudi followed in 1937. My father got out in 1936. They all started off in Chicago, strangely enough. Herbert died in 1994, Rudi in 2017, but Jack is still alive, age 97. Jack is a nobel laureate in physics who befriended me and was my childhood role model, very much responsible for where I went to college and how I've spent my life, but that is another story.

Mark arrived at the University of Chicago in the fall of 1972. He had already made up his mind to work with me, and I never knew whether that was more because of his mathematical interests or because of the family connection. He helped convince others to work in algebraic topology, spearheading a wonderful group of eight people who obtained their PhDs as my advisees in the three years 1977–79.

One of them was Bob Bruner, who wrote me about Mark that "the first thing that comes to mind is his laugh and his ability to see things in a humorous light. Another of them, Jeff Caruso, wrote "I didn't know him very well but in our conversations he was always helpful. He enjoyed explaining things, and helped me to learn about moduli spaces and other topics. I still remember vividly his witty portrayal of Prof. Rothenberg in the 1976 Beer Skit." Mark was charismatic and had a bubbling but caustic sense of humor. He was then still a teenager at heart. He had to be bailed out after one escapade, when he was caught for driving too slowly, but the details are hazy in my memory.

Mark's thesis concerned Dyer Lashof operations on highly structured ring spectra, called E_∞ ring spectra or, in their weaker up-to-homotopy version, H_∞ ring spectra. These operations are homology analogues of the classical Steenrod operations on the cohomology of spaces. In particular, he computed these operations on the homology of Eilenberg-Mac Lane spectra. I foolishly had expected such operations to be trivial, but Mark proved how very wrong I was. His calculations were long ignored, but they have recently made a significant comeback, as have other of his early contributions.

Mark also did his fair share of the relevant foundational work, and it is impossible to talk about his early work without putting it into the general context of a pair of Springer volumes, both published in 1986 but containing mostly work from the late 1970's. One is "H_∞ ring spectra and their applications", by Mark together with Bruner, Jim McClure, and myself [2]. Mark's computational results appear there. One highlight that was long unappreciated gives very general criteria for when a p-local H_2-ring spectrum splits as a wedge of Eilenberg-MacLane spectra $H\mathbf{Z}_p$, or more generally EM spectra $H\mathbf{Z}_{p^r}$, or Brown-Peterson spectra BP. This work has been resurrected with Tyler Lawson's remarkable recent proof [10] (published in 2018) that BP at $p = 2$ is not an E_{12} ring spectrum, answering a long studied question that I asked in 1975. The book as a whole is the starting point of the study of power operations in stable homotopy theory, which now pervade that subject.

The other is "Equivariant stable homotopy theory", by Gaunce Lewis, Mark, and myself, with contributions by McClure [13]. It is no exaggeration to say that this book inititated the study of the subject of its title as a major branch of algebraic topology. Interest in it was resurrected by the extraordinary and unexpected role of equivariant stable homotopy theory in the remarkable solution by Mike Hill, Mike Hopkins, and Doug Ravenel of the Kervaire invariant problem [8] (published in 2016). To quote from Paul Goerss's Mathematics Review of [8] "This paper marks the renaissance and reinvigoration of equivariant stable homotopy theory. While this has been an important subfield since at least the 1970s, the unexpected application of equivariant techniques to such an important problem has brought the study of group actions in stable homotopy theory to the front of the stage. ... The foundation text remains [13]".

Modesty forbids more extensive quotes from Paul's reviews of [8] and of the earlier expository paper [9].[1] A sadly essential point now is that [13] was a large scale collaboration, and both of my principal co-authors are now gone. I was requested to write a Memorial Tribute to Mark, but it is impossible for me to do so without also saying something in appreciation of Gaunce's work: he died, also of brain cancer, on May 17, 2006.[2] Mark and Gaunce were totally different. In contrast to Mark's background, Gaunce was long a teacher at the First United Methodist Church of Oswego, where he served as liturgist coordinator.

Mark was quick and sharp. Gaunce was slower but incredibly methodical. Working together with the two of them was exhilarating. Both were adept at finding my mistakes. Here is an illustrative quote from an introduction to a paper of mine. "I am very grateful to Steinberger for finding the mistakes in [-] and a related mistake in [-]". Gaunce's expertise, especially his remarkable application of Freyd's adjoint functor theorem to construct an adjunction between the prespectra and spectra of [13] is what made the original construction and analysis of the equivariant stable homotopy category possible.

Gaunce continued on with equivariant stable homotopy theory. He brought to it a powerful and unusual blend of categorical and computational thinking. People unfamiliar with modern algebraic topology think of equivariant cohomology with coefficients in an abelian group as $H^*(EG \times_G X; A)$. That is Borel cohomology. While it is powerful and useful, it is only a very special case of Bredon cohomology, the equivariant cohomology theory that satisfies the dimension axiom. With McClure and myself [11], Gaunce introduced $RO(G)$-graded Bredon cohomology. That requires Mackey functor coefficients, and it is now understood to be central to equivariant algebraic topology. For example, for the obvious reason that one cannot embed a G-manifold in any $\mathbb{R}^q$ with trivial G-action, one cannot even make sense of Poincaré duality without $RO(G)$-grading. However, $RO(G)$-graded cohomology is extraordinarily difficult to compute. It is a stroke of luck that the only

[1]It is unfortunate that, as the senior author in many large collaborations, my name is often cited alone, giving me disproportionate credit for joint work.

[2]There was no Notices tribute to him.

actual equivariant calculation in the solution to the Kervaire invariant problems is flukishly easy; the genius is in the reduction to that calculation.

In the papers [14, 6] , Gaunce and his student Kevin Ferland carried out what to this day are some of the most difficult and interesting calculations in equivariant algebraic topology. Gaunce made many other important contributions to that subject. With McClure and me, he proved two necessary steps in the proof of the Segal conjecture and generalized that result to classifying G-spaces [12, 22]. In underappreciated large scale solo papers [17, 20], he made great progress in understanding equivariant stable homotopy theory for incomplete universes, which involves using parts but not all of $RO(G)$; that is closely, but mysteriously, related to recent work by Andrew Blumberg and Mike Hill [1] (published in 2015) that grew out of the solution to the Kervaire invariant problem.

Other solo papers, motivated equivariantly but studied in an illuminating categorical framework for the relevant homological algebra show that standard results, like projective implies flat for modules over a ring, can actually fail in more general contexts [18, 19]. With Halvard Fausk and me, he computed the Picard group, that is the group of unit objects, of the equivariant stable homotopy category [5]. With Mike Mandell, he made a systematic study of the equivariant universal coefficient and Künneth theorems [21]. He also made many valuable contributions to unstable equivariant algebraic topology, especially to the study of the equivariant Hurewicz theorem and the construction of Eilenberg-Mac Lane G-spaces associated to representations of G [16, 15].

In contrast to Gaunce, Mark went in a different direction after his early work in algebraic topology. While his later work was still largely equivariant, it was now in geometric topology. It was done mostly in collaboration with James West [29, 30, 31, 33, 32] and partly also with Sylvain Cappell, Julius Shaneson, and Shmuel Weinberger [4, 3]. I'll let Jim tell the story.

From Jim West. "Mark arrived at Cornell excited about trying to use the extension of simple-homotopy theory to the homeomorphism theory of Hilbert cube manifolds and to ANRs made by work of Tom Chapman, Bob Edwards and me. Mark had a vision to apply it where the "classical" formulation in terms of chain complexes was difficult or impossible to apply. It was particularly adaptable to situations where infinite processes were involved.

We discovered that the equivariant C^0 homeomorphism theory of locally linearizable actions of finite groups on (finite dimensional) manifolds was exactly such a situation. Earlier work by Wu-Chung Hsiang, Tom Chapman, and Steve Ferry had shown that some classical K^0 and K^1 obstructions to smooth or piecewise linear classification were not obstructions to C^0 homeomorphism classification in the inequivariant context.

We had a very fruitful collaboration which involved developing an equivariant surgery theory for locally linearizable actions. I think it would be fair to say that our work really opened this subject for further research. The high points of our research were Mark's Inventiones paper [27] and the joint paper with Cappell and Shaneson in the American Journal titled "Non-linear similarity begins in dimension 6 [4]. I was congratulated very warmly in person on the latter result by Georges De Rham and by Ed Floyd.

In this collaboration, we were definitely equal partners. Mark was the strategist. He was usually the one who came up with the applications that might be accessible using our techniques. Technically, he made all the algebraic equivariant surgery computations, while I concentrated on the controlled infinite processes."

To give a bit more idea of what this is all about, non-linear similarity asks when linearly inequivalent representations of G can be G-homeomorphic,. Analysis of the Picard group of the stable homotopy category is somewhat analogous, since it in part concerns the classification of representation spheres up to G-homotopy type.

Jim does not allude to his first paper with Mark [29], in which they observe that a Serre fibration between CW complexes is a Hurewicz fibration. From the point of view of model categorical foundations for homotopy theory, in which these two kinds of fibrations play vastly different roles, this geometric result is quite curious. Another pair of papers [26, 28], by Mark alone, clarifies the nature of PL fibrations.

Turning to another kind of contribution, Mark deserves sole credit for the creation of an impressive new journal of mathematics, namely the New York Journal of Mathematics (NYJM). Mark founded that in 1994 as the first electronic general mathematics journal, and he was its Editor-in-Chief. Mark wrote an article about it which was published in the January 1996 issue of the Notices and is available on line [25]. Mark was interested in both the economic and the technical benefits of electronic journals. For the former, he, along with many others at the time, was especially concerned about the sometimes exorbitant cost of many mathematics journals, and he saw it as a major advantage of electronic journals that they could be free.[3]

He also hoped that new journals like the NYJM might dent the hold of the top ranked journals. He hated snobbishness in general and the rankings of journals in particular. He would have nodded in agreement with the article [7] about the tyranny of the top five journals in Economics, which could just as well have been written about Mathematics. In fact, as he knew, both [2] and [13] are in large part shotgun marriages of articles unpublishable in top journals at the time, hence their late appearance.

Mark was very much focused on the technical potentials of electronic journals, and he wrote two informative (if perhaps technically dated) articles that focus on the creation of the NYJM [25] and on the existing and potential relevant technology [24]. One interesting technical innovation in the NY Journal can be found at `http://nyjm.albany.edu/search/j/ghindex.html` where one can search all past publications of the NYJM at once for key strings of symbols or words. However, the focus of the journal is on quality and expertise, as its very strong editorial board attests.[4]

I wish I had words to do justice to the personalities of so many friends and colleagues now gone. I can hear Mark laughing at me as I try.

References

[1] Andrew J. Blumberg and Michael A. Hill. Operadic multiplications in equivariant spectra, norms, and transfers. *Adv. Math.*, 285:658–708, 2015.

[2] R. R. Bruner, J. P. May, J. E. McClure, and M. Steinberger. *H_∞ ring spectra and their applications*, volume 1176 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 1986.

[3] I can imagine his snort of laughter if he were told that pdf files of individual chapters of [13] are on sale by the publisher for $29.95 per chapter. (In fairness, an ebook version of the book is $59.99).

[4] A blog by a satisfied author gives a readable description [23].

[3] Sylvain E. Cappell, Julius L. Shaneson, Mark Steinberger, Shmuel Weinberger, and James E. West. The classification of nonlinear similarities over Z_{2^r}. *Bull. Amer. Math. Soc. (N.S.)*, 22(1):51–57, 1990.

[4] Sylvain E. Cappell, Julius L. Shaneson, Mark Steinberger, and James E. West. Nonlinear similarity begins in dimension six. *Amer. J. Math.*, 111(5):717–752, 1989.

[5] H. Fausk, L. G. Lewis, Jr., and J. P. May. The Picard group of equivariant stable homotopy theory. *Adv. Math.*, 163(1):17–33, 2001.

[6] Kevin K. Ferland and L. Gaunce Lewis, Jr. The $RO(G)$-graded equivariant ordinary homology of G-cell complexes with even-dimensional cells for $G = \mathbb{Z}/p$. *Mem. Amer. Math. Soc.*, 167(794):viii+129, 2004.

[7] James Heckman and Sidharth Moktan. The tyranny of the top five journals. Institute for New Economic Thinking, Oct 2, 2018; `https://www.ineteconomics.org/perspectives/blog/the-tyranny-of-the-top-five-journals`.

[8] M. A. Hill, M. J. Hopkins, and D. C. Ravenel. On the nonexistence of elements of Kervaire invariant one. *Ann. of Math. (2)*, 184(1):1–262, 2016.

[9] Michael A. Hill, Michael J. Hopkins, and Douglas C. Ravenel. The Arf-Kervaire problem in algebraic topology: sketch of the proof. In *Current developments in mathematics, 2010*, pages 1–43. Int. Press, Somerville, MA, 2011.

[10] Tyler Lawson. Secondary power operations and the Brown-Peterson spectrum at the prime 2. *Ann. of Math. (2)*, 188(2):513–576, 2018.

[11] G. Lewis, J. P. May, and J. McClure. Ordinary $RO(G)$-graded cohomology. *Bull. Amer. Math. Soc. (N.S.)*, 4(2):208–212, 1981.

[12] L. G. Lewis, J. P. May, and J. E. McClure. Classifying G-spaces and the Segal conjecture. In *Current trends in algebraic topology, Part 2 (London, Ont., 1981)*, volume 2 of *CMS Conf. Proc.*, pages 165–179. Amer. Math. Soc., Providence, R.I., 1982.

[13] L. G. Lewis, Jr., J. P. May, M. Steinberger, and J. E. McClure. *Equivariant stable homotopy theory*, volume 1213 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 1986. With contributions by J. E. McClure.

[14] L. Gaunce Lewis, Jr. The $RO(G)$-graded equivariant ordinary cohomology of complex projective spaces with linear $\mathbf{Z}/p$ actions. In *Algebraic topology and transformation groups (Göttingen, 1987)*, volume 1361 of *Lecture Notes in Math.*, pages 53–122. Springer, Berlin, 1988.

[15] L. Gaunce Lewis, Jr. Equivariant Eilenberg-Mac Lane spaces and the equivariant Seifert-van Kampen and suspension theorems. *Topology Appl.*, 48(1):25–61, 1992.

[16] L. Gaunce Lewis, Jr. The equivariant Hurewicz map. *Trans. Amer. Math. Soc.*, 329(2):433–472, 1992.

[17] L. Gaunce Lewis, Jr. Change of universe functors in equivariant stable homotopy theory. *Fund. Math.*, 148(2):117–158, 1995.

[18] L. Gaunce Lewis, Jr. The category of Mackey functors for a compact Lie group. In *Group representations: cohomology, group actions and topology (Seattle, WA, 1996)*, volume 63 of *Proc. Sympos. Pure Math.*, pages 301–354. Amer. Math. Soc., Providence, RI, 1998.

[19] L. Gaunce Lewis, Jr. When projective does not imply flat, and other homological anomalies. *Theory Appl. Categ.*, 5:No. 9, 202–250, 1999.

[20] L. Gaunce Lewis, Jr. Splitting theorems for certain equivariant spectra. *Mem. Amer. Math. Soc.*, 144(686):x+89, 2000.

[21] L. Gaunce Lewis, Jr. and Michael A. Mandell. Equivariant universal coefficient and Künneth spectral sequences. *Proc. London Math. Soc. (3)*, 92(2):505–544, 2006.

[22] J. P. May and J. E. McClure. A reduction of the Segal conjecture. In *Current trends in algebraic topology, Part 2 (London, Ont., 1981)*, volume 2 of *CMS Conf. Proc.*, pages 209–222. Amer. Math. Soc., Providence, R.I., 1982.

[23] Orr Shalit. Reflections on the New York Journal of Mathematics. Blog; `https://noncommutativeanalysis.com/2012/10/22/reflections-on-the-new-york-journal-of-mathematics`.

[24] Mark Steinberger. The demands on electronic journals in the mathematical sciences. The Journal of Electronic Publishing Dec 1998; `https://quod.lib.umich.edu/j/jep/3336451.0004.208?view=text;rgn=main`.

[25] Mark Steinberger. Electronic mathematics journals. Notices Amer Math Soc Jan 1996; `www.ams.org/notices/199601/steinberger.pdf`.

[26] Mark Steinberger. The classification of PL fibrations. *Michigan Math. J.*, 33(1):11–26, 1986.
[27] Mark Steinberger. The equivariant topological s-cobordism theorem. *Invent. Math.*, 91(1):61–104, 1988.
[28] Mark Steinberger. PL fibrations are fibrations in the PL category. *Topology Appl.*, 29(3):219–222, 1988.
[29] Mark Steinberger and James West. Covering homotopy properties of maps between C.W. complexes or ANRs. *Proc. Amer. Math. Soc.*, 92(4):573–577, 1984.
[30] Mark Steinberger and James West. Equivariant h-cobordisms and finiteness obstructions. *Bull. Amer. Math. Soc. (N.S.)*, 12(2):217–220, 1985.
[31] Mark Steinberger and James West. On the geometric topology of locally linear actions of finite groups. In *Geometric and algebraic topology*, volume 18 of *Banach Center Publ.*, pages 181–204. PWN, Warsaw, 1986.
[32] Mark Steinberger and James West. Approximation by equivariant homeomorphisms. I. *Trans. Amer. Math. Soc.*, 302(1):297–317, 1987.
[33] Mark Steinberger and James West. Equivariant handles in finite group actions. In *Geometry and topology (Athens, Ga., 1985)*, volume 105 of *Lecture Notes in Pure and Appl. Math.*, pages 277–295. Dekker, New York, 1987.