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Errata for
Rings, Modules, and Algebras in Stable Homotopy Theory
by A. D. Elmendorf, I. Kriz, M. A. Mandell, and J.P. May
with an appendix by M. Cole

Welcome to this errata. It was up to date on October 30, 2000.

Please send any errors or comments on EKMM to mandell@math.uchicago.edu

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February 4, 1997

Error: Lemma II.6.1 is false as stated and has incorrect proof.

This was noticed by J. Dolan, University of California, Riverside, who gave the following counterexample (I have rephrased it, any errors due to me):

Let $\mathbb{S}$ be the monad on sets that takes a set to its disjoint union with the set $\{1, 2\}$. Let $\mathbb{T}$ be the monad on the category of (sets under the set $\{1, 2\}$) that: (i) if 1 and 2 map to the same point p, then $\mathbb{T}$ takes the disjoint union with the set $\{3\}$, and regards the result as a set under $\{1, 2\}$ with 1 and 2 both going to the point p; (ii) if 1 and 2 map to different points, then $\mathbb{T}$ does nothing. The multiplication for this monad is the map that identifies the two new points "3" in case (i) and the identity map in case (ii) (this is a natural transformation since a a case-(i) set cannot map to a case-(ii) set). The category of algebras over the monad $\mathbb{TS}$ is the category of sets under $\{1, 2\}$ but the category of $\mathbb{T}$-algebras in (sets under $\{1, 2\}$) is the category (sets under $\{1, 2\}$ with another chosen point if 1 and 2 map to the same point).

Problem: The map in the middle of p. 45, $\mathbb{T}Q \to \mathbb{TS}Q \to Q$, does not make any sense because the unit $Q \to \mathbb{S}Q$ is not a map of $\mathbb{S}$-algebras.

Correction: The lemma can be corrected by adding the extra hypothesis that $\mathbb{T}$ preserves reflexive coequalizers of $\mathbb{S}$-algebras. The rest of the book is unaffected since this extra hypothesis holds in all cases when II.6.1 is applied.

Lemma II.6.1. *Let $\mathbb{S}$ be a monad in a category $\mathscr{C}$ and let $\mathbb{T}$ be a monad in the category $\mathscr{C}[\mathbb{S}]$ of $\mathbb{S}$-algebras.* If $\mathbb{T}$ preserves reflexive coequalizers in $\mathscr{C}[\mathbb{S}]$, *then the category $\mathscr{C}[\mathbb{S}][\mathbb{T}]$ of $\mathbb{T}$-algebras in $\mathscr{C}[\mathbb{S}]$ is isomorphic to the category $\mathscr{C}[\mathbb{TS}]$ of algebras over the compound monad $\mathbb{TS}$ in $\mathscr{C}$. Moreover, the unit of $\mathbb{T}$ defines a map $\mathbb{S} \to \mathbb{TS}$ of monads in $\mathscr{C}$. An analogous assertion holds for comonads.*

Proof. Proceed as on p. 45 until reaching the problem. For a $\mathbb{TS}$-algebra Q construct the map $\mathbb{T}Q \to Q$ as follows.

Note that the following diagram is a reflexive coequalizer of $\mathbb{S}$-algebras.

$$\mathbb{SS}Q \underset{\mathbb{S}\xi}{\overset{\mu}{\rightrightarrows}} \mathbb{S}Q \xrightarrow{\xi} Q$$

where μ is the multiplication for $\mathbb{S}$ and ξ is the $\mathbb{S}$-algebra action on Q, i.e. $\xi = \omega \circ \eta$ where ω is the given $\mathbb{TS}$-algebra action on Q and η is the unit of $\mathbb{T}$. By the assumption on $\mathbb{T}$, the following diagram is also a reflexive coequalizer of $\mathbb{S}$-algebras.

$$\begin{tikzcd} \mathbb{TSS}Q \arrow[r, shift left, "\mathbb{T}\mu"] \arrow[r, shift right, "\mathbb{TS}\xi"'] & \mathbb{TS}Q \arrow[r, "\mathbb{T}\xi"] & \mathbb{T}Q \end{tikzcd}$$

We obtain the map $\chi\colon \mathbb{T}Q \to Q$ from the map $\omega\colon \mathbb{TS}Q \to Q$ by showing that

$$\mathbb{T}\mu \circ \omega = \mathbb{TS}\xi \circ \omega.$$

Note that if χ exists, it is clearly a $\mathbb{T}$-action map and satisfies $\omega = \xi \circ \mathbb{T}\chi$. The existence of χ follows from considering the composite of the map $\mathbb{TS}\eta_{\mathbb{S}Q}\colon \mathbb{TSS}Q \to \mathbb{TSTS}Q$ with the associativity diagram for $\omega\colon \mathbb{TS}Q \to Q$,

$$\begin{tikzcd} \mathbb{TSS}Q \arrow[r, "\mathbb{TS}\eta"] & \mathbb{TSTS}Q \arrow[r] \arrow[d] & \mathbb{TS}Q \arrow[d] \\ & \mathbb{TS}Q \arrow[r] & Q \end{tikzcd}$$

and realizing that the square in the following diagram commutes.

$$\begin{tikzcd} \mathbb{TSS}Q \arrow[r, "\mathbb{TS}\eta_{\mathbb{S}Q}"] \arrow[d, "\mathbb{T}\mu"'] & \mathbb{TSTS}Q \arrow[d, "\xi_{\mathbb{TS}Q}"] & \\ \mathbb{TS}Q \arrow[r, "\mathbb{T}\eta_{\mathbb{S}Q}"'] & \mathbb{TTS}Q \arrow[r] & \mathbb{TS}Q \end{tikzcd}$$

The square commutes because it is obtained by applying the functor $\mathbb{T}$ to the diagram asserting that the unit of $\mathbb{T}$ is a map of $\mathbb{S}$-algebras, specialized to the object $\mathbb{S}Q$. The right-most horizontal arrow is the multiplication on $\mathbb{T}$, and so the compsite map $\mathbb{TSTS}Q \to \mathbb{TS}Q$ is by definition the multiplication of $\mathbb{TS}$.

February 11, 1998

Error: Corollary I.3.3 asserts that the unit and counit of the Σ, Ω adjunction is a homotopy equivalence for tame spectra, but only proves that it is a space-wise homotopy equivalence.

This was noticed by M. Cole, University of Michigan.

Correction: Insert the word "space-wise" before the phrase "homotopy equivalence". This does not affect the rest of the arguments in the book, since only the fact that the map is a weak equivalence is ever used.

October 3, 2000

Error: Some of the assertions on p. 85 (§IV.5) in the proof of the convergence of the spectral sequences of Theroem IV.4.1 are incorrect.

This was reported by L. G. Lewis, Syracuse University, in summer 1999.

Correction: Let $M_\infty = \operatorname{Colim} M_p$. Then $\pi_* M_\infty = 0$, the map $M \to M_\infty$ is an h-cofibration, and M_∞/M is a cell R-module. Thus, M_∞/M is a cell R-module approximation of ΣM that comes with a filtration by subcell complexes, namely M_p/M. The spectral sequences in Theorem IV.4.1 are up to a shift and sign the spectral sequences of the induced filtrations on $(M_\infty/M) \wedge_R N$ and $F_R(M_\infty/M, N)$, and so convergence is standard (see J. M. Boardman, "Conditionally convergent spectral sequences" in *Homotopy invariant algebraic structures (Baltimore, MD, 1998)*, 49–84, Contemp. Math., 239, Amer. Math. Soc., Providence, RI, 1999). I apologize for the delay in adding this correction to the errata.

October 3, 2000

Error: The proof (on p. 85–86, §IV.5) of Proposition IV.4.4 is incorrect.

This was noticed by A. Lazarev, University of Bristol.

Comment: Presumably, this pairing arises from an appropriate filtration on

$$F_R(M', N) \wedge_R F_R(L', M'),$$

where L' and M' are cell approximations of L and M (or of ΣL and ΣM); however, this is a technically difficult argument to make, and so at the current time I offer **NO CORRECTION**. Note also that there is a typo in the statement of Proposition IV.4.4: The "S" and "S^*" should be "R" and "R^*".

October 30, 2000

Error: Theorem X.2.9 needs the extra assumption that K_* is tame in the spectrum case. It is correct as stated in the R-module case.

Correction: The proof is correct as written with this extra assumption, using I.3.5 to see that the cofiber sequences are cofiber sequences in the stable category. The extra assumption is not needed in the R-module case because of I.6.5.