THE LIST OF COURSES, 2003

1. DISCRETE MATHEMATICS (weeks 1–8)

Laszlo Babai

The course covers topics in number theory, combinatorial structures, linear algebra and discrete probability, finite groups, the theory of algorithms and combinatorial models in the theory of computing. The course will highlight surprising interactions among these areas. Students will discover each field through solving sequences of challenging problems. A number of open problems will be discussed.

The course will be divided into two modules. The first module (weeks 1–4) will focus on the interaction between linear algebra, combinatorics, and algorithms.

The second module (weeks 5–8) will focus on combinatorial and algorithmic aspects of finite groups.

The two modules will be sufficiently independent that if you missed the first module, you can still join the second. Returning students will not be bored.

PQ: Consent of instructor. CS-17400 (Discrete Math) or CS-27000 (Algorithms) helpful but not required. Basic linear algebra and finite fields desirable. Interested students are encouraged to take Math-28400, a.k.a. CS-27400, Honors Combinatorics and Probability, offered in Spring, see
http://www.cs.uchicago.edu/courses/descriptions.php.

2. KNOTS AND LINKS (weeks 1–4)

Benson Farb (1–2), Jeffrey Brock (3–4)

Abstract: How can we tell when a loop in space is knotted? How can we tell one knot from another? How knotted is a "random" loop in space? How can we tell when two (or more) loops in space are linked?

In this course we will address these and many other questions. In order to do so we will use tools from a number of areas of mathematics. Topics might include: fundamental groups, Reidemeister moves, knot projections, Seifert surfaces and the Alexander polynomial. We also hope to explore some still unkown questions (like the third question above), collecting evidence via computer and other experiments.

3. INTRODUCTION TO GROUPS AND GEOMETRY (weeks 1–2)

Diane Herrmann

This course is intended as a condensed introduction to group theory via geometry and symmetry. We will begin with the symmetries of regular polygons, develop the basics of group theory and work toward problems to work on to prepare for work in the YSP geometry courses. Topics may also include symmetries of regular polyhedra, symmetries of infinitely repeating patterns, and generators and relations.

4. INVITATION TO PROBABILITY THEORY (week 1)

Robert Fefferman

This will be a series of lectures about the most basic ideas of this theory: sample spaces, probability of events, random variables, expectation and variance, and independence. We shall discuss the laws of large numbers and present some beautiful applications of these results such as the approximation of continuous functions by polynomials. Finally, we shall discuss the central limit theorem and such related topics as characteristic functions and Brownian motion.

5. TOPICS IN ODE'S (week 2)
Eduard Kirr
This will give basic background material that will be used in the following course.

6. MATHEMATICS IN INDUSTRIAL APPLICATIONS (weeks 3–4)
Fadil Santosa
This problem-solving session involves mathematical modeling, analysis and computer simulation. Students will break up into two teams and work on a problem for the two-week period. The first problem has to do with path planning for multiple vehicles (on-land and flying). The vehicles must avoid each other as well as static obstacles put in their way. The mathematics involved include ordinary differential equations and optimal control. The second problem concerns the design of opthalmic lenses. The goal is to assign power correction to different areas of the lens while minimizing the undesirable effects of astigmatism. The mathematics involved include differential geometry of surfaces in addition to optimization.
The teams will be supervised by the instructor who will act as project manager and consultant. Basic mathematical background will be provided prior to the project but during the project's duration, student will learn other techniques on-the-job similar to what occurs in an industrial research environment. Regular progress reports will be expected, and a final presentation is expected in the eighth week.

7. FOLLOW UP ON AN INTIVATION TO PROBABILITY THEORY: MARKOV CHAINS, MARTINGALES, AND MORE ... (weeks 5–6)
Peter Constantin
We will start with a quick introduction to the theory of discrete-time Markov chains. We will discuss also Markov processes with continuous time but discrete state space (such as Poisson processes). We will discuss then discrete time martingales, Markov (stopping) times, convergence theorems and inequalities for discrete time martingales. If time permits we will talk a little about continuous time martingales and Brownian motion.

8. FINITE TOPOLOGICAL SPACES (weeks 5–7)
Peter May
There are some strange and intriguing papers in the literature that show that finite simplicial complexes, which are the familiar polyhedrally decomposed spaces, can in fact be approximated "for all purposes of algebraic topology" by finite topological spaces, that is finite sets with suitable topologies. At first sight, this seems quite counterintuitive. Nevertheless, it is not too hard to prove. We shall explain the arguments, introducing many basic ideas in homotopy theory along the way. No previous knowledge of topology, algebraic or otherwise, will be assumed. There is lots of unexplored territory here. For example, how can one describe models for surfaces as finite topological spaces?

9. INTRODUCTION TO TOPOLOGICAL DEGREE IN EUCLIDEAN SPACES (weeks 7–8)
Marta Lewicka
This course aims to provide a self-contained introduction to the theory of topological degree (the Brouwer degree) in Euclidean spaces. It is intended for students

most interested in analysis and topology. We will define the Brouwer degree using analytic techniques, prove its basic properties, and apply it to several classical theorems, such as the Brouwer fixed point theorem, the Poincaré-Bohl theorem, the fundamental theorem of algebra, the odd-mapping theorem, the antipodal theorem, the Lusternik-Schnirelmann theorem, and the ham sandwich theorem. Many exercises and problems will be offered.

10. STUDENT PRESENTATIONS (week 8)