SHORT ABSTRACTS: 2008 SUMMER VIGRE REU

All concepts mentioned in the abstracts will be carefully defined

APPRENTICE PROGRAM

Apprentices attend courses offered in the full program during week 1, and a morning program of introductory material will also be given. Apprentices are also welcome to attend all courses in later weeks. Several of the courses in the full program are designed to be accessible to apprentice participants.

Weeks 2–4: Laci Babai and Miklos Abert

LINEAR ALGEBRA AND THE SPECTRAL THEORY OF GRAPHS

The course will develop the usual topics of linear algebra and illustrate them on highly unusual and often striking applications to combinatorics. Emphasis will be on creative problem solving and discovery. The basic topics include permutations, determinants, linear transformations, the characteristic polynomial, Euclidean spaces, orthogonalization, the Spectral Theorem. The principal application area to be highlighted is the connection between the combinatorial structure and the spectrum of graphs, including expansion, random walks, chromatic number, and more.

FULL PROGRAM

Whatever faculty have posted in past abstracts, the material actually presented has generally diverged from that originally planned so far in advance. Therefore, only quite abbreviated indications are being posted.

1. Laci Babai. Topics in discrete mathematics. Weeks 1 – 8.
2. Miklos Abert. Problem solving course. Weeks 2 – 5
3. Benson Farb. Topics in geometry. Weeks 1–4.
4. Gregory Lawler. Topics in probability. Weeks 5 – 6
5. Peter May, Vigleik Angeltveik, and Alissa Crans. Finite topological spaces and related topics. Weeks 1 – 8.

1. DISCRETE MATHEMATICS

Weeks 1 – 8: Laci Babai

The course will cover gems in a number of areas of mathematics; topics will be built up through solving sequences of challenging problems. Observing striking interactions between seemingly distant areas will be the central theme. Topics will include graph theory, combinatorics, discrete geometry, number theory, group theory, analysis (orthogonal polynomials), finite models in the theory of computing. Linear algebra, multivariate polynomials, and discrete probability will be central to the methodology.

Advance knowledge of the subject areas listed is not required.

Returning students will not be bored. The overlap with Honors Combinatorics and Probability (Math-274, CMSC-284) will be minimal.

PQ: Consent of instructor. Basic linear algebra. CMSC-17400 (Discrete Math) helpful but not required.

2. PROBLEM SOLVING COURSE

Weeks 2 – 5: Miklos Abert

This year I am organizing a problem-solving course again (last time was three years ago). Problems range from fun exercises through beauty contest winners to honest research starters. Topics will include algebra, real analysis, topology, discrete math and number theory. Students of all levels are welcome - the course is structured in a way to make sure that everyone can participate. Minicourses will be given to help catching up in some topics.

3. PROBABILITY AND THE HEAT EQUATION

Weeks 5 – 6: Gregory Lawler

The flow of heat can be understood very well by imagining a very large number of "heat particles" moving independently. These heat particles do random walks (or Brownian motions). One can also study heat flow without probability using a partial differential equation called (surprisingly!) the heat equation. I will discuss the relationship between probabilistic and nonprobabilistic approaches to the study of heat and discuss some of the classical ways to solve the heat equation and the Dirichlet problem (equilibrium heat distribution). Important in both approaches are the Laplacian and harmonic functions (and in probability, the related notion of a martingale). We will consider both discrete time and space (which involves diagonalization of symmetric matrices) and continuous time and space (which leads to Fourier series).

Previous exposure to probability is not required.

4. TOPOLOGICAL METHODS IN GROUP THEORY

Weeks 1 – 4: Benson Farb

Abstract: The goal of this course is to explain how topology is a powerful tool for studying the algebraic/combinatorial structure of infinite groups.

5. FINITE TOPOLOGICAL SPACES AND RELATED TOPICS

Weeks 1 – 8: Peter May, Vigleik Angeltveit, and Alissa Crans

There is a fascinating and little known theory of finite topological spaces. We will present lots of the basic theory and how it relates to partially ordered sets, simplicial complexes, and finite groups. For example, we shall reinterpret a basic unsolved problem in finite group theory in terms of finite topological spaces. We will go slowly enough that all can follow (promise!!!), but the material is sure to be new to even the most advanced students.