

MORE IS BETTER

AN INVESTIGATION OF MONOTONICITY ASSUMPTION IN ECONOMICS

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Abstract

Monotonicity of preference is assumed in the conventional economic theory to arrive at important conclusions such as “more commodities are preferred to less.” However, Gary Becker claims that monotonicity is not necessary for these conclusions in his book *Economic Theory* [1]. This paper investigates Becker’s claim.

1 The Conventional Theory¹

Economists assume that economic agents (often referred to as the consumer in the context of utility maximization) attempt to maximize utility (which means satisfaction in vernacular).

Definition 1.1. Let C denote a set of all possible spending arrangements. Consumption bundles, c , are elements of this set.

Utility, U , can be represented as a function² from C to \mathbb{R} .

Definition 1.2. \succsim is a preference relation that denotes weak preference. If c_1 is weakly preferred to c_2 then $c_1 \succsim c_2$. In other words, $U(c_1) \geq U(c_2)$.

\succ is a preference relation that denotes strong preference. If c_1 is strongly preferred to c_2 then $c_1 \succ c_2$. In other words, $U(c_1) > U(c_2)$.

Definition 1.3. \sim is a preference relation that denotes indifference. If $c_1 \succsim c_2$ and $c_1 \precsim c_2$ are both true then the consumer is said to be indifferent. In other words, $U(c_1) = U(c_2)$

With these definitions, one can now examine the so-called axioms of choice.

Definition 1.4. Reflexibility For any bundle c_i , $c_i \succsim c_i$

¹See Mas-Colell *et al.* [2] for more complete exposition of microeconomic theory.

²There are many utility functions that are not continuous. I will only consider the continuous variety in this paper.

Definition 1.5. Completeness Given two bundles, one of the following is true:

1. $c_1 \succsim c_2$
2. $c_2 \succsim c_1$
3. $c_1 \sim c_2$

Completeness implies that the consumer can judge between any two consumption bundles.

Definition 1.6. Transitivity If $c_1 \succsim c_2$ and $c_2 \succsim c_3$ then $c_1 \succsim c_3$

Transitivity makes the completeness axiom applicable to any number of bundles. The consumer is said to be rational if his or her preference is transitive and complete.

Definition 1.7. (Strict) Monotonicity³ Suppose there are n commodities in both c_1 and c_2 . Monotonicity means that if c_1 contains more of some or all commodities, but no less of any, than c_2 ($c_1 \geq c_2$) then $c_1 \succsim c_2$.

Strict monotonicity means that if c_1 contains more of each commodity than c_2 ($c_1 > c_2$) then $c_1 \succ c_2$

Now let us consider the consequences that arise from monotonicity of preference.

Definition 1.8. A function U is non-decreasing if $c_1 \geq c_2$ implies $U(c_1) \geq U(c_2)$. A non-increasing utility function is defined similarly.

A function U is strictly increasing if $c_1 > c_2$ implies $U(c_1) > U(c_2)$. A strictly decreasing utility function is defined similarly.

Theorem 1.1. *Preferences are monotone if and only if U is non-decreasing and they are strictly monotone if and only if U is strictly increasing.*

Proof. First, we prove that the preference relation \succsim can be represented by a utility function. Then it becomes obvious that preferences are monotone if and only if U is non-decreasing.

Utility functions are real valued functions defined on C . Therefore for any $c_1, c_2 \in C$, either $U(c_1) \geq U(c_2)$ or $U(c_1) \leq U(c_2)$ is true (1). If a utility function represents the \succsim then (1) implies that $c_1 \succsim c_2$ or that $c_2 \succsim c_1$ (2). (2) is true because of completeness.

We now extend this result to any $c \in C$. Suppose $c_1 \succsim c_2$ and $c_2 \succsim c_3$. If a utility function represents \succsim then $U(c_1) \geq U(c_2)$ and $U(c_2) \geq U(c_3)$. Therefore, $U(c_1) \geq U(c_3)$ which implies that $c_1 \succsim c_3$ (3). (3) is true because of transitivity.

It has now been proven that the preference relation \succsim can be represented by a utility function (and also that the consumer needs to be rational for the said representation to happen).

The second part of the theorem is proven similarly. □

It is now clear that the monotonicity assumption implies that the consumer derives more “pleasure” from consuming more commodities (or more precisely, the consumer would not be “saddened” by the prospect of consuming a greater quantity of commodities). Consider constraints

³A more general assumption of nonsatiation can be made *in lieu* of the monotonicity assumption—it is essentially monotonicity defined in the language of limits. There always exists some bundle preferred to the bundle being consumed which may or may not be affordable.

the consumer would face when he or she attempts to maximize his or her utility. Expressed mathematically, the utility maximization problem is: $\max_{x_1, \dots, x_n} U = U(x_1, \dots, x_n)$ s.t. $I \geq \sum_{i=1}^n p_i x_i$ where p_i is the price of the i th commodity, x_i , and I is the consumer's income⁴. If there were no constraints, the consumer would continue to consume more commodities because of monotonicity. And because of monotonicity, the solution to the utility maximization problem ends up at the boundary of the opportunity set ($I = \sum_{i=1}^n p_i x_i$). In other words, the consumer would fully spend his or her income (whilst the consumer would still prefer to consume beyond his or her income).

Definition 1.9. An indifference curve is the collection of all bundles of same utility; in other words, $U(x_1, \dots, x_n) = \bar{U}$. Where \bar{U} is some fixed utility.

The more technical, and therefore less interesting, consequence of monotonicity assumption is that the indifference curves cannot intersect.

Theorem 1.2. *Indifference curves cannot intersect.*

Proof. Consider two indifference curves, I_1 and I_2

Suppose the said indifference curves intersect and let c_1 be the bundle at the intersection, c_2 the bundle on I_1 and c_3 the bundle on I_2 .

Since c_2 and c_3 are on different indifference curves, $U(c_2) > U(c_3)$ or $U(c_2) < U(c_3)$ is true. In turn, either $c_2 \succ c_3$ or $c_2 \prec c_3$ is true.

At the same time, c_2 is on the same indifference curve with c_1 and c_3 on the same curve with c_1 . That is, $c_1 \sim c_2$ and $c_1 \sim c_3$. Since the preference relation is assumed to be monotone $c_1 \sim c_2$ and $c_1 \sim c_3$ cannot hold simultaneously—contradiction. \square

This concludes the very brief discussion of conventional economic theory. Armed with the knowledge of conventional theory, we can now investigate Becker's claim.

⁴The utility maximization problem can therefore be solved using constrained optimization methods such as the Lagrangian method.

2 Becker's Claim

In *Economic Theory*, Gary Becker claims that monotonicity is *implied* by the theory. Indeed, he only lists economic rationality as the necessary assumption for the mathematical treatment of utility maximization. Becker proves by contradiction why more is preferred to less.⁵

If less of any good were preferred to more, and if the good had a non-negative price, a consumer would increase his utility by reducing his demand for that good until he either consumed none of it or preferred more to less. If a consumer were “satiated”—indifferent between more and less of all goods—and if work were “irksome,” he would reduce his hours worked and thus his income until either his income (or at least his earnings) vanished or he preferred more of some goods to less; he would then consume only these goods in positive quantities. It is this sense that utility theory implies that more is preferred to less of all goods actually consumed.

Essentially Becker is not considering the consumer's income as given but as the consequence of his or her labor. Therefore the “Beckerian” utility maximization problem can be stated as:

$$\begin{aligned} \max_{x_1, \dots, x_n} U &= U(x_1, \dots, x_n) \\ \text{s.t. } \sum_{i=1}^n p_i x_i &\leq I = Y + w \cdot t, t + l = T \end{aligned}$$

where Y is non-earned income, w hourly wage, t hours worked, l leisure (hours spent not working) and T total time endowment.⁶

In this model, it is clear that the consumer would always operate on the boundary of his or her opportunity set. If an interior bundle is preferred then the consumer would decrease the hours spent working thereby causing the line $\sum_{i=1}^n p_i x_i = I = Y + wt$ to move towards the origin until it touches the desired bundle.

Another point needs to be discussed. We saw that indifference curves do not intersect because of the monotonicity assumption—does the initial exclusion of monotonicity allow for the possibility of intersecting indifference curves?

This is a very important question. If indifference curves can cross then the utility function does not accurately represent the consumer's preference—the consumer cannot compare different bundles of commodities.

⁵Note even if the consumer decides to save some of his or her income, the solution to his or her utility maximization problem will still be on the boundary if the savings are defined to be a “commodity.” Also consider the commodities that the sane would actually prefer less to more (say, deadly poison)—defining a commodity to be “lack” of that commodity would ensure the boundary solution.

⁶Becker's proof is merely the interpretation of the Beckerian utility maximization problem's first order conditions.

Theorem 2.1. *Indifference curves cannot intersect (even without the monotonicity assumption).*

Proof. Consider two indifference curves, I_1 and I_2 .

Suppose the said indifference curves intersect and let c_1 be the bundle at the intersection, c_2 the bundle on I_1 and c_3 the bundle on I_2 .

Since c_2 and c_3 are on different indifference curves, $U(c_2) > U(c_3)$ or $U(c_2) < U(c_3)$ is true. In turn, either $c_2 \succ c_3$ or $c_2 \prec c_3$ is true.

c_1 is on the same indifference curve with c_2 and with c_3 . $c_1 \sim c_2$ and $c_1 \sim c_3$ are both true and by transitivity $c_2 \sim c_3$ —a contradiction. \square

3 Conclusion

In the conventional economic theory, the assumption of monotonicity leads to three implications: (1) “more is better”; (2) the solution to the utility maximization problem lies on the boundary of the opportunity set; and (3) indifference curves cannot intersect.

Becker’s formulation of the utility maximization problem excludes monotonicity assumption. Even though monotonicity is never assumed, the three implications of the assumption can still be derived.

The Beckerian theory is stronger mathematically as it arrives at the same conclusion with fewer assumptions. Also a case can be made that the Beckerian theory also mimics the real world more closely than the conventional variety since Becker defines income not as a mere constraint but as a function of work—only those of us who are incredibly lucky can count income as given.

4 References

- [1] Becker, G. S.: *Economic Theory*, Transaction Publishers, New Brunswick, N.J., 2008.
- [2] Mas-Colell, A., Whinston, M. D., Green, J. R.: *Microeconomic Theory*, Oxford University Press, New York, N.Y., 1995.