MAY/ANNO PROBLEMS, WEDNESDAY JUNE 23

1. Enumerate the monoids with three elements and determine their group completions.

2. Enumerate the Abelian monoids with four elements and determine their group completions.

3. Try to carry out the first construction of a group completion in the non-Abelian case. Be careful with the order of multiplication. Why doesn't this work?

4. Let $N \subset M$ be the set of elements $n \in M$ such that $i(n) = e$ in $G(M)$. Describe this set explicitly when M is Abelian. What if M is non-Abelian?

5. [Research problem] Is there an Abelianization functor, Ab say, from monoids to Abelian monoids, that satisfies the expected universal property: For any Abelian monoid A and homomorphism of monoids $f\colon M \to A$, there is a unique map of Abelian monoids $\tilde{f}\colon Ab(M) \to A$ such that $\tilde{f} \circ i = f$, where $i\colon M \to Ab(M)$ is Abelianization.

Hint: for the novice, explore an explicit Abelianization construction. For the categorically sophisticated, determine whether Freyd's adjoint functor theorem applies.