# THE HEXATONIC SYSTEMS UNDER NEO-RIEMANNIAN THEORY: AN EXPLORATION OF THE MATHEMATICAL ANALYSIS OF MUSIC

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Abstract. Neo-Riemannian theory developed as the mathematical analysis of musical trends dating as far back as the late 19th century. Specifically, the musical trends at hand involved the use of unorthodox sets of pitches. These sets had the characteristic of retaining their triadic harmonies, but of sounding atonal. Neo-Riemannian theory not only allowed for the mathematical construction and description of these sets, some of which later were called the Hexatonic Systems, but also a means to explain how these systems were used. Involving the application of the $PLR$ Operations, the analysis of the Hexatonic Systems is a primary component of neo-Riemannian theory. In this paper, I give a self-contained description of Rick Cohn's Hexatonic Systems, as well as the group-theoretic structures at hand.

## Contents



## 1. Introduction

Neo-Riemannian theory was initiated by David Lewin in the late 20th century, reviving the theory originally established by Hugo Riemann, a music theorist of the late 19th century, after whom the theory is named. Riemann developed a system to relate triads to each other, and, continued by Lewin, this system was expanded to allow for a more complete description of triadic sets via the $P$, $L$, and $R$ operations. Adding to this theory, Rick Cohn developed the Hexatonic Systems, sets

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of pitches whose mathematical and musical construction involved $PLR$ manipulation of triadic strucutres. As a major focus of neo-Riemannian theory, the $P$, $L$, and $R$ operations will be a crucial component of this paper. More specifically, this paper will explore the mathematical derivation of the Hexatonic Systems through the $PLR$ operations, focusing greatly on the mathematical implications involved in this derivation.

There are four Hexatonic Systems, each of which denotes a set of six pitch classes. In music, these systems manifest themselves as collections of pitches such as scales and chords. In the analysis of music, Hexatonic Systems often sound atonal despite their triadic harmonies. Hence, because of this atonal quality, they are not accessible through traditional, tonal analyses. Recognizing Hexatonic Systems under neo-Riemannian theory, however, allows one to understand the relationship between pitches. Generally, it helps to identify relationships under a Hexatonic System or between Hexatonic Systems where, under other means, no relationships seem to exist.

Mathematically, the Hexatonic Systems are derived by the application of the $L$ and $P$ operations to the set of major and minor triads. The mathematical components of the Hexatonic Systems which I will focus on deal mainly with the $PLR$ operations and the groups that they generate.

## 2. The Basics

To understand the Hexatonic Systems under neo-Riemannian theory, it is necessary to establish some pre-requisite knowledge. This knowledge is primarily a knowledge of music: namely, pitch and the major and minor triads. These topics, however, will be explained within a mathematical context.

### 2.1. **Pitch.**

Pitch is a fundamental concept in music. Simply defined, the term *pitch* refers to the specific frequency of a sound. Though there are many frequencies and corresponding pitches, it is through a specific grouping of pitches that we structure music. Much of Western music, for example, is based on the major scale, a special subset of the 12-tone system. Briefly recalled now, this system is founded on 12 fundamental pitches. Though there are more than 12 pitches, every pitch of this system is a whole number of octaves from these 12 fundamental pitches.

To understand what an octave is mathematically, it is useful and common to arrange pitches on a logarithmic scale base 2, where the unit for this scale is an octave. In other words, an octave is such that if pitch-1 has twice or half the frequency as that of pitch-2, then pitch-1 and pitch-2 are octaves of each other.

From this system, we define an equivalence relation on pitches by saying that two pitches are equivalent if they differ by an integer amount of octaves. The equivalence classes which result from this equivalence relation are called the pitch classes.

In the 12-tone system, there are 12 commonly used pitch classes which divide an octave into 12 equal parts on a logarthmic scale. These pitch classes can be seen in the following.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | A#<br>B♭ | B | C | C#<br>D♭ | D | D#<br>E♭ | E | F | F#<br>G♭ | G | G#<br>A♭ | A |

The pitch classes are named by a combination of letters, sharps (#), and flats (♭). The interval between each adjacent pitch class is called a *half-step* or *semitone*. Pitches with two names are called *enharmonically equivalent* pitches.

2.2. **The Integer Model of Pitch.** The *integer model of pitch* arises from the redefinition of pitch classes to integers modulo 12, where the pitch class $C$ is taken as 0 and can be seen as follows.

| A | A# B♭ | B | C | C# D♭ | D | D# E♭ | E | F | F# G♭ | G | G# A♭ | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 10 | 11 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Addition and subtraction is modulo 12, thus identifying the pitches with $\mathbb{Z}_{12}$. For example, $10 + 3 = 1$ mod12, and $2 - 3 = 11$ mod12. Modular arithmetic can be used to determine musical intervals between pitch classes. For example, the musical interval between $F$ and $D$ is $5 - 2 = 3$. An interval of 3 is described as an interval of 3 semitones, or half-steps. Furthermore, modular arithmetic allows for a mathematical definition of musical transposition and inversion.

**Definition 2.1.** Musical *transposition*, $T_n$, is mathematical translation such that

$$T_n : \mathbb{Z} \to \mathbb{Z}$$
$$T_n(x) = x + n.$$

**Definition 2.2.** Musical *inversion*, $I_n$, is mathematical reflection such that

$$I_n : \mathbb{Z} \to \mathbb{Z}$$
$$I_n(x) = -x + n.$$

Transposition and inversion map a pitch, a pitch-class, or, more importantly, a triad to any another pitch, pitch-class, or triad. Transposition and inversion will be necessary later.

2.3. **The Major and Minor Triads.** Through the integer model of pitch, the major and minor triads can also be defined mathematically. A triad is an ordered set of three pitches which are sequentially called the 'root', the 'third', and the 'fifth'. A *major triad* is an ordered set of three pitches in which the third is four semitones above the root, and the fifth is seven semitones above the root. The $C$ major triad, for example, is $C$, $E$, $G$, or $\{0, 4, 7\}$. A *minor triad*, on the other hand, is an ordered set of three pitches in which the third is three semitones above the root, and the fifth is seven semitones above the root. The $c$ minor triad, for example, is $C$, $E\flat$, $G$, or $\{0, 3, 7\}$.
The order of the triads does not matter when playing the triads aloud. However, the ordering that is written is helpful for establishing clear definitions under the mathematical system in use. Note that a triad is conventionally named after its root. Also note that, by convention, major triads are associated with upper-case letters, whereas minor triads are associated with lower-case letters. This can be shown in the following table of all the major and minor triads.

| Major Triads | Minor Triads |
|---|---|
| $C = \langle 0,4,7\rangle$ | $\langle 0,3,7\rangle = c$ |
| $C\#, D\flat = \langle 1,5,8\rangle$ | $\langle 1,4,8\rangle = c\#, d\flat$ |
| $D = \langle 2,6,9\rangle$ | $\langle 2,5,9\rangle = d$ |
| $D\#, E\flat = \langle 3,7,10\rangle$ | $\langle 3,6,10\rangle = d\#, e\flat$ |
| $E = \langle 4,8,11\rangle$ | $\langle 4,7,11\rangle = e$ |
| F=$\langle 5,9,0\rangle$ | $\langle 5,8,0\rangle = f$ |
| $F\#, G\flat = \langle 6,10,1\rangle$ | $\langle 6,9,1\rangle = f\#, g\flat$ |
| $G = \langle 7,11,2\rangle$ | $\langle 7,10,2\rangle = g$ |
| $G\#, A\flat = \langle 8,0,3\rangle$ | $\langle 8,11,3\rangle = g\#, a\flat$ |
| $A = \langle 9,1,4\rangle$ | $\langle 9,0,4\rangle = a$ |
| $A\#, B\flat = \langle 10,2,5\rangle$ | $\langle 10,1,5\rangle = a\#, b\flat$ |
| $B = \langle 11,3,6\rangle$ | $\langle 11,2,6\rangle = b$ |

## 3. The PLR Operations

Having defined the integer model of pitch and the major-minor triads, we can now move on to neo-Riemannian theory. Specifically, we will be dealing with the $P$, $L$, and $R$ operations.

The $PLR$ operations are bijections which map a major triad to a minor triad, and vice versa.

**Definition 3.1.** The *parallel* operation, $P$, maps a major triad to its parallel minor triad, and vice versa. A major triad's parallel minor has a third which is one semitone below the third of the major triad. A minor triad's parallel major has a third which is one semitone above the third of the minor triad. All other pitches between parallel triads are the same.

**Example 3.2.** An example of the $P$ operation is as follows:
$P(C) = c$ and $P(c) = C$
or $P\langle 0,4,7\rangle = \langle 0,3,7\rangle$ and $P\langle 0,3,7\rangle = \langle 0,4,7\rangle$.

**Definition 3.3.** The *leading tone exchange* operation, $L$, maps a major triad to the minor triad whose fifth is one semitone below the root of the major triad. It maps a minor triad to the major triad whose root is one semitone above the fifth of the minor triad. All other pitches between leading tone exchange triads are the same.

**Example 3.4.** An example of the $L$ operation is as follows:
$L(C) = e$ and $L(e) = C$
or $L\langle 0,4,7\rangle = \langle 4,7,11\rangle$ and $L\langle 4,7,11\rangle = \langle 0,4,7\rangle$.

**Definition 3.5.** The *relative* operation, $R$, maps a major triad to its relative minor triad, and vice versa. The relative minor of a major triad has a root that is two semitones above the major triad's fifth. The relative major of a minor triad has a fifth which is two semitones below the root of the minor triad. All other pitches between relative triads are the same.

**Example 3.6.** An example of the $R$ operation is as follows:
$R(C) = a$ and $R(a) = C$
$R\langle 0,4,7\rangle = \langle 9,0,4\rangle$ and $R\langle 9,0,4\rangle = \langle 0,4,7\rangle$.

The $PLR$ operations generate the group known as the $PLR$-group. The $PLR$-group consists of all composites of the $P$, $L$, and $R$ operations. It is helpful to consider the $PLR$ operations as certain permutations of the set of major and minor triads, and the $PLR$-group consequently as a subgroup of all permutations of the set of major and minor triad. Through this particular sense of the $PLR$ operations and the $PLR$-group, it is logical to look at the operations separately as subgroups of the $PLR$-group. The group $\langle L, P\rangle$, the $LP$-group, is a subgroup of the $PLR$-group that consists of all the composites of $L$ and $P$. When $\langle L, P\rangle$ acts on the set of all major and minor triads, orbits of the triads are produced. The pitch classes which make up the triads of each orbit are called a *Hexatonic System*. There are four orbits generated by $\langle L, P\rangle$, and hence there are four Hexatonic Systems. To demonstrate the derivation of these systems, however, we must first define group actions.

**Definition 3.7.** A *group action* of a group $G$ on a set $X$ is a function $G \times X \to X$, or $(g, x) \mapsto gx$, such that

(1) $ex = x \ \forall x \in X$ where $e$ is the identity element of G
(2) $(g_1, g_2)x = g_1(g_2x)$.

**Lemma 3.8.** *If $G$ is a group and $X$ is a set, then a group action of $G$ on $X$ may be defined as a group homomorphism $h : G \to Sym(X)$.*

**Definition 3.9.** If $G$ acts on $X$, the *orbit* of $x \in X$ is $orbit(x) = \{gx \,|g \in G\}$.

## 4. The Hexatonic Systems

The group $\langle L, P\rangle$ is the subgroup of the $PLR$-Group generated by $L$ and $P$, and contains all composites of $L$ and $P$. The orbits produced when $\langle L, P\rangle$ acts on the set of major-minor triads are each composed of six triads. The pitch classes which make up each set of these six triads are the Hexatonic Systems. This construction of the Hexatonic Systems was conceptually developed by Richard Cohn, as can be seen in [2]. It is important to note that a Hexatonic System is not an orbit, but the set of pitch classes which make up the triads of an orbit. It is also important to note that the Hexatonic Systems are not disjoint.

**Example 4.1.** The derivation of the Hexatonic Systems can be shown as follows. $G = \langle L, P\rangle \subseteq Sym(X)$ where $X$ is the set of all major and minor triads. We assume now that $\langle L, P\rangle$ has six distinct elements: $id_X$, $P$, $LP$, $PLP$, $LPLP$, $PLPLP$. This is assumed to demonstrate the calculation of the Hexatonic Systems, but it will be proved later that $\langle L, P\rangle$ does indeed have six elements. Note that $L^2 = id_X = P^2$. Also note that $id_X$ is used instead of $e$ to represent the identity element. This is because $e$ is used to represent the minor chord, $e$.
Thus, applying these six members to the $C$ major triad, we calculate the orbit of the $C$ major triad under $\langle L, P\rangle$:
$id_XC = C$
$PC = c$
$LPC = Lc = A\flat$
$PLPC = PA\flat = a\flat$
$LPLPC = La\flat = E$
$PLPLPC = PE = e$
Therefore, $orbit(C) = \{C, c, A\flat, a\flat, E, e\}$.

The pitches underlying $orbit(C)$ are called a Hexatonic System. Therefore, $Hex(C) = \{0, 3, 4, 7, 8, 11\}$.
Continuing this application to the rest of the triads, it can be shown that only four Hexatonic Systems exist.

**Lemma 4.2.** *If $y \in orbit(x)$, then $orbit(x) = orbit(y)$. (This is standard and will not be proved here).*

**Theorem 4.3.** *There are only four orbits produced when the group $G = \langle L, P\rangle$ acts on the set of major-minor triads; hence, there are only four Hexatonic Systems.*

*Proof.* Direct computation shows that $orbit(C) = \{C, c, A\flat, a\flat, E, e\}$. The resulting Hexatonic System is $Hex(C) = \{0, 3, 4, 7, 8, 11\}$.

Continuing this method of computation through the major triads to C#, $orbit(C\#) = \{C\#, c\#, A, a, F, f\}$ by the following calculation:
$id_X C\# = C\#$
$PC\# = c\#$
$LPC\# = Lc\# = A\flat$
$PLPC\# = PA\flat = a\flat$
$LPLPC\# = La\flat = E$
$PLPLPC\# = PE = e$
And the resulting Hexatonic System is $Hex(C\#) = \{1, 4, 5, 8, 9, 0\}$.

Continuing through the major triads to D, $orbit(D) = \{D, d, B\flat, b\flat, F\#, f\#\}$ by the following calculation:
$id_X D = D$
$PD = d\#$
$LPD = Ld\# = B\flat$
$PLPD = PB\flat = b\flat$
$LPLPD = Lb\flat = F\#$
$PLPLPD = PF\# = f\#$
And the resulting Hexatonic System is $Hex(D) = \{2, 5, 6, 9, 10, 1\}$.

Continuing through the major triads to D#, $orbit(D\#) = \{D\#, d\#, B, b, G, g\}$ by the following calculation:
$id_X D\# = D\#$
$PD\# = d\#$
$LPD\# = Ld\# = B$
$PLPD\# = PB = b$
$LPLPD\# = Lb = G$
$PLPLPD\# = PG = g$
And the resulting Hexatonic System is $Hex(D\#) = \{3, 6, 7, 10, 11, 2\}$.

It is unnecessary to continue from here. Each orbit calculated thus far has six members, representing all twenty-four of the major and minor triads without repition. This is evident from the fact that the orbits are pairwise disjoint: $orbit(C) \cap orbit(C\#) = \emptyset$, $orbit(C\#) \cap orbit(D) = \emptyset$, etc. Thus, because each major and minor triad is included in these four orbits, by Lemma 3.2, continuing to apply $\langle L, P\rangle$ through the triads will only yield one of the four orbits already calculated.

So, there can only be four orbits. Thus, by definition, there must only be four Hexatonic Systems, as each Hexatonic System is based on an orbit. □

**Theorem 4.4.** *The group $\langle L, P\rangle$ has six elements.*

*Proof.* The elements $id_X C$, $PC$, $LPC$, $PLPC$, $LPLPC$, and $PLPLPC$ are each a member of the group $\langle L, P\rangle$, and they are all different by the computations shown above. This implies, by Remark 4.4, that $id_X$, $P$, $LP$, $PLP$, $LPLP$, and $PLPLP$ are all different. Therefore, $\langle L, P\rangle$ has at least six members, or $|\langle L, P\rangle| \geq 6$.

For permutations longer than these six configurations, we observe three things:

1. Observation: $L^2 = id_X = P^2$. Consequently, only alternating words will be significant.

2. Observation: $L(PLPLP)C = C$ by direct computation. It is known that $gT_n = T_n g$ and $I_n g = gI_n$ $\forall$ $g \in \langle P, L, R\rangle$ and $\forall$ $n \in \mathbb{Z}_{12}$ [1]. It is also known that every major chord is of the form $T_n C$ for some $n \in \mathbb{Z}_{12}$, and every minor chord is of the form $I_n C$ for some $n \in \mathbb{Z}_{12}$ [1]. It can therefore be shown that $L(PLPLP) = id_X$:
Applying $T_n$ to $L(PLPLP)C = C$, we get
$T_n L(PLPLP)C = T_n C$
$\Rightarrow L(PLPLP)T_n C = T_n C\ \forall\ n \in \mathbb{Z}_{12}$
$\Rightarrow L(PLPLP) = id_X$ for the major chords.
Similarly for the minor chords:
Applying $I_n$ to $L(PLPLP)C = C$, we get
$I_n L(PLPLP)C = I_n C$
$\Rightarrow L(PLPLP)I_n C = I_n C\ \forall\ n \in \mathbb{Z}_{12}$
$\Rightarrow L(PLPLP) = id_X$ for the minor chords.
Therefore, $L(PLPLP) = id_X$ for all major and minor chords under any transposition or inversion

3. Observation: $LC = e = PLPLPC$ by direct computation. This implies that $L = PLPLP$

From these three observations, we conclude that any (alternating) permutation longer than $L(PLPLP)$ will be a repetition of the six original elements: $id_X C$, $PC$, $LPC$, $PLPC$, $LPLPC$, and $PLPLPC$. We also conclude that any permutation ending in $L$ is equivalent to a permutation ending in $PLPLP$, which is equivalent to the permutations already considered.
$\Rightarrow \langle L, P\rangle$ must have at most six elements, or $|\langle L, P\rangle| \leq 6$.

Therefore, because $|\langle L, P\rangle| \geq 6$ and $|\langle L, P\rangle| \leq 6 \Rightarrow |\langle L, P\rangle| = 6$. □

**Example 4.5.** The sets of pitch classes encompassed by the Hexatonic Systems have been used throughout music, even before this mathematical formulation of the Hexatonic Systems was developed. One example is the augmented scale: $C$, $D$#, $E$, $G$, $G$#, $B$. The augmented scale is the Hexatonic System underlying the orbit of the $C$ major triad. The augmented scale can be cited in classical music such as Franz Liszt's *Die Legende vom heiligen Stanislaus* [2] from the early nineteenth

century, and in jazz music from the twentieth century.
Identifying Hexatonic Systems in a piece of music allows one to better understand the progression of pitches underneath the Hexatonic System and the progression of pitches from one Hexatonic System to another.

## 5. Further Inquiry

5.1. **The Group $\langle L, P\rangle$ is Isomorphic to $S_3$.** As shown in Section 4, $\langle L, P\rangle$ is a group of order 6. Consequently, it can be determined as to whether $\langle L, P\rangle$ is isomorphic to the only other groups of order 6. It is well known that every group of order 6 is either $\mathbb{Z}_6$ or $S_3$, where $S_3$ is the group of permutations on three elements. Because $\mathbb{Z}_6$ is abelian and $S_3$ is not abelian, we need only to check if $\langle L, P\rangle$ is abelian.

**Definition 5.1.** A group $G$ under the operation $\bullet$ is *abelian* if it is commutative. That is,

$$\forall\ a, b \in G,\ a \bullet b = b \bullet a$$

**Theorem 5.2.** *The group $G = \langle L, P\rangle$ is isomorphic to $S_3$.*

*Proof.* The group $G = \langle L, P\rangle$ is a group under the operation: function composition.
$G$ is isomorphic to $S_3$ if $G$ is not abelian.
Assume $G$ is abelian.
This implies that for $P, L \in G$, $PL = LP$.
Given $L = PLPLP$ and $PL = LP \Rightarrow L = LPPLP$.
Because $P^2 = id_X = L^2$, $L = LPPLP$ can be written as $L = Lid_XLP = LLP = id_XP = P$.
Thus, $L = P$, which is a contradiction.
Therefore, $G$ is not commutative and, obviously, not abelian.
Thus, $G$ is isomorphic to $S_3$. □

5.2. **The Groups $\langle L, R\rangle$ and $\langle P, R\rangle$.** Besides $\langle L, P\rangle$, the other subgroups of $\langle P, L, R\rangle$ that can further be considered are $\langle L, R\rangle$ and $\langle P, R\rangle$.
In [1], it is proved that the $PLR$ group is equal to $\langle L, R\rangle$, so in this case we obtain the chromatic system $\mathbb{Z}_{12}$.
When the group $\langle P, R\rangle$ acts on the set of major and minor triads, the *Octatonic Systems* are produced. The same method used to compute the Hexatonic Systems can be used to derive the Octatonic Systems. Only the results will be shown here. No theorems or proofs about the Octatonic Systems will be given.

**Example 5.3.** The derivation of the Octatonic Systems can be shown as follows.
$G = \langle P, R\rangle \subseteq Sym(X)$ where $X$ is the set of all major and minor triads. We assume that $\langle P, R\rangle$ has eight elements: $id_X$, $P$, $RP$, $PRP$, $RPRP$, $PRPRP$, $RPRPRP$, and $PRPRPRP$. Note that $R^2 = id_X = P^2$.
Then, by applying $id_X$, $P$, $RP$, $PRP$, $RPRP$, $PRPRP$, $RPRPRP$, and $PRPRPRP$ to the $C$ major triad, we calculate the orbit of the $C$ major triad under the group $\langle P, R\rangle$:
$id_XC = C$
$PC = c$
$RPC = Rc = D\#$
$PRPC = PD\# = d\#$
$RPRPC = Rd\# = F\#$

$PRPRPC = PF\# = f\#$
$RPRPRPC = Rf\# = A$
$PRPRPRPC = PA = a$
Therefore, $orbit(C) = \{C, c, D\#, d\#, F\#, f\#, A, a\}$, and $Oct(C) = \{0, 1, 3, 4, 6, 7, 9, 10\}$.

Continuing through the major triads to $C\#$, we calculate the orbit of the $C\#$ major triad under the group $\langle P, R\rangle$:
$id_X C\# = C\#$
$PC\# = c\#$
$RPC\# = Rc\# = E$
$PRPC\# = PE = e$
$RPRPC\# = Re = G$
$PRPRPC\# = PG = g$
$RPRPRPC\# = Rg = A\#$
$PRPRPRPC\# = PA\# = a\#$
Therefore, $orbit(C\#) = \{C\#, c\#, E, e, G, g, A\#, a\#\}$, and $Oct(C\#) = \{1, 2, 4, 5, 7, 8, 10, 11\}$.

Continuing through the major triads to $D$, we calculate the orbit of the $D$ major triad under the group $\langle P, R\rangle$:
$< P, R >$:
$id_X D = D$
$PD = d$
$RPD = Rd = F$
$PRPD = PF = f$
$RPRPD = Rf = G\#$
$PRPRPD = PG\# = g\#$
$RPRPRPD = Rg\# = B$
$PRPRPRPD = PA\# = b$
Therefore, $orbit(D) = \{D, d, F, f, G\#, g\#, A, a\}$, and $Oct(D) = \{2, 3, 5, 6, 8, 9, 11, 0\}$. Notice that the orbits found in this example are pairwise disjoint, suggesting that all 24 of the major-minor triads are included.

These are the familiar octatonic scales found in Arabic music for centuries, as well as Jazz and other 20th century music. Hence, their recent mathematical development is especially interesting.

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