CALCULUS 153: MIDTERM 1

Please answer all questions in a blue book that's provided to you (even the true/false). Don't forget to write your name. There are two sides to this exam.

Problem 1 (16 points). Determine the least upper bound and greatest lower bound of the following sets, or state that they do not exist. You do not need to justify your answer (4 points each).

(1)
$$(\pi, \infty),$$

(2)
$$\{x : |x - 2| < 1\},$$

(3)
$$\left\{\sin(\frac{\pi}{2n}) : n = 1, 2, \ldots\right\},$$

(4)
$$\{.9, .99, .999, \ldots\}.$$

Problem 2 (21 points). For each of the following sequences, determine whether the sequence converges or diverges. If it converges, find its limit. Show your work. (7 points each).

(1)
$$a_n = (2n)^{1/n};$$

(2)
$$b_n = \cos\left(\pi + \frac{\ln n}{n}\right).$$

(3)
$$c_n = \frac{3^n}{n!}\cos(\ln n).$$

Problem 3 (21 points). Find the following limits. Show your work (7 points each).

(1)
$$\lim_{x\to 0} \frac{\sin(2x) + x^2 - 2x}{e^x - 1 - x},$$

(2)
$$\lim_{x\to\infty} \cos\left(\frac{1}{x}\right)^x,$$

(3)
$$\lim_{x\to\infty} \ln(x + 1) - \ln x.$$

Problem 4 (8 points). State the ϵ – K definition of $\lim_{n\to\infty} a_n = L$.

Problem 5 (14 points). Prove that if $a_n \to L$ and $b_n \to M$ then $a_n + b_n \to L + M$.

Problem 6 (20 points). Please answer true or false (you do not need to justify your answer).

(1) If M is an upper bound for S, and $M \in S$, then M is the least upper bound for S.
(2) The least upper bound of a set S must be contained in S.
(3) The least upper bound of a set of irrational numbers must be irrational.
(4) A sequence that is bounded and increasing must converge.
(5) If $a_n + b_n$ converges then both a_n and b_n must converge.