

CALCULUS 133: SAMPLE MIDTERM 2

Problem 1 (10 points). Evaluate each integral, or show that it diverges (5 points each):

(1)

$$\int_0^{\infty} \frac{x}{1+x^2} dx.$$

(2)

$$\int_1^2 \frac{x}{\sqrt{x^2-1}} dx.$$

Problem 2 (9 points). For each of the following sequences, determine whether the sequence converges or diverges. If it does, find its limit (3 points each).

(1) $a_n = \frac{n^2+2}{4n^2-3}$.

(2) $b_n = (-1)^n \frac{1}{n^2}$.

(3) $c_n = \frac{n}{\ln n}$.

Problem 3 (8 points).

(1) Use the integral test to show that

$$\sum_{k=1}^{\infty} k e^{-k}$$

converges (5 points).

(2) Now prove it using the ratio test (3 points).

Problem 4 (16 points). Using the test or method of your choice, find whether the following series converge or diverge. Please show the details; do not simply write down the answer (4 points each).

(1)

$$\sum_{k=1}^{\infty} \frac{k^2}{1+k^3},$$

(2)

$$\sum_{k=1}^{\infty} \frac{k}{2^k},$$

(3)

$$\sum_{k=1}^{\infty} \ln(k)^2,$$

(4)

$$\sum_{k=2}^{\infty} \frac{2}{k \ln k}.$$

Problem 5 (7 points). *Brad and Angelina divide up a cake as follows. Brad takes a third of the cake and Angelina takes a half. There is now a sixth left. Brad takes a third of that and Angelina takes half. There is now one thirty-sixth left. Brad agains takes a third and Angelina a half. Suppose this continues on indefinitely. What portion of the cake does Angelina end up getting?*