

QUESTIONS.

1. Measurable sets form a σ -algebra.
2. An example of a non-measurable set.
3. $\limsup_n f_n$ is measurable if f_n are.
4. The Egorov theorem.
5. Bounded convergence theorem.
6. Monotone convergence theorem.
7. Lebesgue dominated convergence theorem.
8. The Vitali lemma.
9. Derivative of a monotone function exists a.e.
10. A BV function is a difference of two monotone functions.
11. Derivative of an anti-derivative.
12. A function is an anti-derivative if and only if it is absolutely continuous.
13. If μ is a radon measure then $\mu(A) = \inf\{\mu(U), A \subset U, U \text{ is open}\}$.
14. The Caratheodory criterion for a measure to be Radon.
15. Existence of a continuous extension of a continuous function from a compact set to \mathbb{R}^n .
16. The Lusin theorem.
17. Fatou's lemma.
18. L^1 -convergence implies a.e. convergence along a subsequence.
19. The Fubini theorem.
20. Let U be an open set and $\delta > 0$. Show that there exists a countable collection \mathcal{G} of disjoint balls with diameter less than δ so that $|U \setminus \bigcup_{B \in \mathcal{G}} B| = 0$.
21. Let μ and ν be Radon measures. Show that $D_{\mu\nu}$ exists and is finite μ -a.e. and is μ -measurable.
22. The Lebesgue decomposition theorem.
23. The Radon-Nikodym theorem.
24. The Lebesgue-Besikovitch theorem.
25. A Lebesgue measurable function is approximately continuous a.e.
26. Hahn decomposition.
27. The Riesz representation theorem in L^p , $1 \leq p < \infty$.
28. Weak compactness of Radon measures.
29. The Riemann-Lebesgue lemma.
30. The Dini and Jordan criteria.
31. The de-Bois-Raymond theorem.
32. Summability of the Cesaro averages in L^p .
33. The Fourier inversion formula for \mathcal{S} .
34. The dyadic maximal function is weak (1,1) and the limit exist a.e.
35. The Calderon-Zygmund decomposition.
36. The Kolmogorov theorem: the Hilbert transform is weak (1,1).
37. The Marcinkiewicz theorem.
38. The Riesz theorem for the Hilbert transform.
39. Anything else you think you should know.