TEST #1 REVIEW QUESTIONS

MATH 151 SECTION 35, FALL 2005

The first test will be in class on Friday, October 21st. Many of the questions will be taken from the following list, perhaps modified. Some questions will also be taken from the homework, so be sure you understand how to do any homework problems you got wrong. You don't need to turn in solutions to these problems—they're only to help you prepare for the test—but we will go over answers in the problem session on Thursday, October 20th.

Problem 1. Solve the following inequalities. Write your solutions using interval notation.

(a) $x(x-1)(x+2) \leq 0$
(b) $|2-2x| < 4$

Problem 2. For each of the following sets, write an inequality using absolute values whose solution is the given set.

(a) $(-1.2, -0.8)$
(b) $(-1, 2) \cup (2, 5)$

Problem 3. Let A and B be real numbers and suppose that for every real number $\varepsilon > 0$, we have $|A - B| < \varepsilon$. Prove that in fact $A = B$. *Hint: Use proof by contradiction. Assume $A \neq B$ and consider what happens if $\varepsilon = \frac{1}{2}|A-B|$.*

Problem 4. For each function and specified x-value, give the limit if it exists (you do not have to prove it). If the limit does not exist, briefly explain why not.

(a)
$$f(x) = \frac{x^3 - 3x}{x+3}, \qquad x \to 3$$

(b)
$$f(x) = \frac{1}{(x-2)^2}, \qquad x \to 2$$

(c)
$$f(x) = \begin{cases} 0 & \text{if } x \text{ is an integer} \\ x^2 & \text{otherwise} \end{cases}, \qquad x \to -1$$

Problem 5. Write down the opposite (i.e. the negation) of the following statements. (Don't just write "not" in front of them!)

(a) Every month in Chicago includes at least one day below freezing.
(b) There is at least one season of the year in which there are no days when it rains in Chicago.
(c) In every year, there is at least one week during which it snows every day in Chicago.

Problem 6. **NOTE: This question has a 100% probability of being on the test.** Write out the formal (i.e. ε-δ) definition of the statement

$$\lim_{x \to c} f(x) = L.$$

Problem 7. Give formal proofs (using the ε-δ definition) of the following limit statements.

(a)
$$\lim_{x \to 1} (5 - 2x) = 3$$

(b)
$$\lim_{x \to 2} (x^2 + 3x - 1) = 9$$

(c)
$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3} = 4$$

Problem 8. Joe Confused has given the following "proof" that $\lim_{x \to 3} (2x - 1) = 5$.

> "Suppose $0 < |x - 3| < \delta$. Thus $|2x - 6| = 2|x - 3| < 2\delta$, so pick $\varepsilon = 2\delta$. Then we have $|(2x - 1) - 5| < \varepsilon$ as desired."

Briefly explain his error.