

REVIEW QUESTIONS FOR TEST #2

MATH 151 SECTION 35, FALL 2005

The second test will be on Friday, November 18th. Many of the questions will be modified versions of those on this list; some may also be taken from the homework. You don't need to turn in solutions to these problems, but we will discuss them in the problem session on Thursday, November 17th.

Problem 1. Compute the derivatives of the following functions.

(a) $f(x) = 3x^2 + x - 4$

(b) $f(x) = \frac{\sqrt{x}-1}{x^2+1}$

(c) $f(x) = \sqrt{3-2x^2}$

(d) $f(x) = \cos(\frac{1}{2}x^2)$

Problem 2. Write out the formal definition (using limits) of when a function f is differentiable at the point x , and if it is differentiable, what its derivative is.

Problem 3. Using the limit definition of the derivative, find the derivatives of the following functions, **and** say where they are differentiable.

(a) $f(x) = 5x^2 - 10x + 1$

(b) $f(x) = \sqrt{x}$

Problem 4. For each of the following graphs of functions $f(x)$, draw a possible graph of the derivative $f'(x)$.

(a)

(b)

(c)

Problem 5. Use the intermediate value theorem to show that there exists an x with $0 \leq x \leq \frac{\pi}{2}$ such that $\cos x = x$.

Problem 6. Find an equation for the line tangent to the graph of $f(x) = x^3 + 3x^2 - 1$ at $x = 1$.

Problem 7. True or False?

- (a) If f is differentiable at x , then f is continuous at x .
- (b) If f is continuous at x , then f is differentiable at x .
- (c) If f is continuous on $[a, b]$ and $f(a) < K < f(b)$, then there exists at least one c with $a < c < b$ such that $f(c) = K$.
- (d) If f is continuous on $[a, b]$, then f is bounded on $[a, b]$.
- (e) If f is continuous on (a, b) , then f attains a maximum value on (a, b) .
- (f) If f and g are differentiable at x , then their product $f \cdot g$ is also differentiable at x .
- (g) If f and $f \cdot g$ are differentiable at x , then g is also differentiable at x .

Problem 8. Suppose I drop a rock from a cliff and it hits the ground at a speed of 100 m/s . (You may approximate $g \approx 10 \text{ m/s}^2$.)

- (a) How much time did the rock spend in the air?
- (b) How high was the cliff?

Problem 9. Let P be a polynomial. Recall that a real number a is a root of P precisely when there exists a polynomial r such that $P(x) = (x - a)r(x)$ (the “factor theorem”). We say that a is a *double root* of P if there exists a polynomial q such that

$$P(x) = (x - a)^2 q(x) \quad \text{and} \quad q(a) \neq 0.$$

- (a) Show that if a is a double root of P , then $P(a) = 0$ and $P'(a) = 0$ but $P''(a) \neq 0$.
- (b) Show the converse: that if $P(a) = 0$, $P'(a) = 0$, and $P''(a) \neq 0$, then a is a double root of P .