Definitions of Limits

There are many different types of limits, each with its own formal definition, but the definitions are all very similar. Here I have tried to emphasize the similarities in form. All the variation arises from essentially two changes: x can approach c from the left, right, or from both sides, and either x or $f(x)$ can go "to infinity" instead of to some finite number.

$\lim_{x\to c} f(x) = L$	means	for every $\varepsilon > 0$	there exists a $\delta > 0$	such that whenever $0 < \lvert x-c\rvert < \delta$,	$\lvert f(x) - L\rvert < \varepsilon$.
$\lim_{x\to c^+} f(x) = L$	means	for every $\varepsilon > 0$	there exists a $\delta > 0$	such that whenever $0 < x-c < \delta$,	$\lvert f(x) - L\rvert < \varepsilon$.
$\lim_{x\to c^-} f(x) = L$	means	for every $\varepsilon > 0$	there exists a $\delta > 0$	such that whenever $-\delta < x-c < 0$,	$\lvert f(x) - L\rvert < \varepsilon$.
$f(x) \to \infty$ as $x \to c$	means	for every $M > 0$	there exists a $\delta > 0$	such that whenever $0 < \lvert x-c\rvert < \delta$,	$f(x) > M$.
$f(x) \to -\infty$ as $x \to c$	means	for every $M < 0$	there exists a $\delta > 0$	such that whenever $0 < \lvert x-c\rvert < \delta$,	$f(x) < M$.
$f(x) \to \infty$ as $x \to c^+$	means	for every $M > 0$	there exists a $\delta > 0$	such that whenever $0 < x-c < \delta$,	$f(x) > M$.
$f(x) \to \infty$ as $x \to c^-$	means	for every $M > 0$	there exists a $\delta > 0$	such that whenever $-\delta < x-c < 0$,	$f(x) > M$.
$f(x) \to -\infty$ as $x \to c^+$	means	for every $M < 0$	there exists a $\delta > 0$	such that whenever $0 < x-c < \delta$,	$f(x) < M$.
$f(x) \to -\infty$ as $x \to c^-$	means	for every $M < 0$	there exists a $\delta > 0$	such that whenever $-\delta < x-c < 0$,	$f(x) < M$.
$\lim_{x\to \infty} f(x) = L$	means	for every $\varepsilon > 0$	there exists a $K > 0$	such that whenever $x > K$,	$\lvert f(x) - L\rvert < \varepsilon$.
$\lim_{x\to -\infty} f(x) = L$	means	for every $\varepsilon > 0$	there exists a $K < 0$	such that whenever $x < K$,	$\lvert f(x) - L\rvert < \varepsilon$.
$f(x) \to \infty$ as $x \to \infty$	means	for every $M > 0$	there exists a $K > 0$	such that whenever $x > K$,	$f(x) > M$.
$f(x) \to -\infty$ as $x \to \infty$	means	for every $M < 0$	there exists a $K > 0$	such that whenever $x > K$,	$f(x) < M$.
$f(x) \to \infty$ as $x \to -\infty$	means	for every $M > 0$	there exists a $K < 0$	such that whenever $x < K$,	$f(x) > M$.
$f(x) \to -\infty$ as $x \to -\infty$	means	for every $M < 0$	there exists a $K < 0$	such that whenever $x < K$,	$f(x) < M$.