REVIEW SHEET #1

MATH 152 SECTION 35, WINTER 2006

The first test will be on Friday, January 27, at the usual class time and place. No books, notes, or calculators will be allowed. Below are a collection of questions designed to help you review for the test. Many of the questions on the test will be modified version of these or problems from the homework.

Problem 1. Calculate the following limits. For limits that do not exist, say if the function becomes arbitrarily large positive ($\to \infty$) or arbitrarily large negative ($\to -\infty$).

(a) $\displaystyle \lim_{x\to\infty} \frac{1}{x^2-1}$

(b) $\displaystyle \lim_{x\to-\infty} \frac{4x^3+x-5}{x^3+3}$

(c) $\displaystyle \lim_{x\to4} \frac{1}{x^2-8x+16}$

(d) $\displaystyle \lim_{x\to0^-} \frac{x}{x^3+10x}$

Problem 2. Calculate the following indefinite integrals.

(a) $\displaystyle \int (x^2-2x+4)\,dx$

(b) $\displaystyle \int \left(\frac{x^4-1}{x^2}\right)\,dx$

Problem 3. Find $f(x)$ given the following information about it.

(a) $f'(x) = 1 - x^2$ and $f(3) = 2$.
(b) $f''(x) = \sin x$, $f'(0) = 0$, and $f(0) = 0$.

Problem 4. Prove (using the formal definition of infinite limits) that $\dfrac{1}{(x+1)^2} \to \infty$ as $x \to -1$.

Problem 5. Find all the extrema (local, endpoint, and absolute) of the function $f(x) = (x - \sqrt{x})^2$.

Problem 6. Find all the interesting points on the graph of the following functions, including intercepts, asymptotes, extrema, vertical tangents, concavity, and inflection points, and sketch graphs of them.

(a) $f(x) = \dfrac{2x^2}{x+1}$

(b) $f(x) = x - x^{1/3}$

Problem 7. Draw possible graphs of functions $f(x)$ satisfying the following conditions.

(a) f has a vertical asymptote at $x = 0$ and a vertical tangent line at $x = 2$.
(b) f has points of inflection at $x = -1$ and $x = 1$, and also $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$.
(c) f is concave up on infinitely many intervals and concave down on infinitely many intervals.
(d) f has three local maxima but no absolute maximum and no absolute minimum.

Problem 8. Prove that if f is an increasing function, then for any function g, the functions g and $f \circ g$ have local extrema at the same values of x.

Problem 9. A tapestry which is 7 feet high hangs on a wall 9 feet above eye level. How far from the wall should an observer stand so that the angle the tapestry subtends at his or her eye is maximal? *(Hint: by Problem 8, an angle θ is maximal when $\tan\theta$ is maximal. You may also want to use the trigonometric formula for $\tan(\alpha - \beta)$.)*

Problem 10. A ball is thrown with an initial velocity of v_0 at an angle of θ from the horizontal. (So if $\theta = \pi/2$, it is thrown straight up.) What value of θ will cause the ball to travel the farthest possible distance before it falling back to the ground? *(Hint: The initial horizontal velocity will be $v_0 \cos\theta$ and the initial vertical velocity will be $v_0 \sin\theta$.)*