REVIEW SHEET #2

MATH 152 SECTION 35, WINTER 2006

The second test will be on Friday, February 24, at the usual class time and place. No books, notes, or calculators will be allowed. Below are a collection of questions designed to help you review for the test. Many of the questions on the test will be modified version of these or problems from the homework.

Problem 1. Suppose that g is a function with the following properties.

(i) g is differentiable everywhere;
(ii) $g'(x) < 0$ for $x < 1$;
(iii) $g'(x) > 0$ for $x > 1$;
(iv) $g'(1) = 0$; and
(v) $g(1) = 0$.

Define a function as follows:

$$G(x) = \int_0^x g(t)dt.$$

Explain why G has the following properties.

(a) G is twice differentiable;
(b) G is concave down for $x < 1$ and concave up for $x > 1$;
(c) $x = 1$ is a critical number of G;
(d) G is increasing everywhere;
(e) G has no local extrema.

Problem 2. Evaluate the following integrals.

(a) $\displaystyle\int \sin(2\pi x)dx$

(b) $\displaystyle\int \sin^3 x \cos x dx$

(c) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx$ *(Hint: let $x = \sin u$.)*

Problem 3. Sketch the region bounded by the curves and find its area.

(a) $y = \sin x$ and $y = \cos x$ between $x = \pi/2$ and $x = 3\pi/2$.
(b) $y = \sqrt[3]{x}$ and $y = x$, in the first quadrant.

Problem 4. Consider the region under the graph of $f(x) = \frac{1}{x}$ between $x = 1$ and $x = B$ for some $B > 1$; call this region Ω_B.

(a) Rotate Ω_B around the x-axis and calculate the volume of the resulting solid in terms of B. Call this volume V_B.
(b) Calculate $\lim_{B\to\infty} V_B$. Notice that it is finite. (Thus, the "infinite solid" obtained by rotating Ω_∞ around the x-axis still has "finite volume".)

Problem 5. Let Ω be the region in the first quadrant bounded by the x-axis, the curve $y = x^2$, and the line $x = 2$. Find the volume of the solid produced by rotating Ω about the y-axis.

Problem 6. True or false?

(a) If f is continuous on $[a, b]$, then f is integrable on $[a, b]$.
(b) If f is continuous on $[a, b]$, then there is at least one number c in (a, b) such that $\int_a^b f(x)dx = (b - a)f(c)$.
(c) If $L_f(P) \leq I \leq U_f(P)$ for all partitions P of an interval $[a, b]$, then $\int_a^b f(x)dx = I$.
(d) If f is integrable, then for any a, b, and c, we have $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$.
(e) If f is continuous on $[a, b]$, then f has an antiderivative on (a, b).
(f) For any function f and any partition P of an interval $[a, b]$, we have $L_f(P) \leq U_f(P)$.
(g) If F and G are antiderivatives of the same function f, then $F(x) = G(x)$.

Problem 7. Let $f(x) = \frac{1}{x}$ and consider the partition $P = \{1, \frac{3}{2}, 2\}$ of the interval $[1, 2]$.

(a) Calculate $U_f(P)$.
(b) Conclude that $\ln 2 < 1$ and therefore $e > 2$.

Problem 8. Show that if two continuous functions f and g have the same average value on every interval, then in fact $f = g$.

Hint #1: Recall from last quarter that if h is continuous and $h(c) > 0$ for some c, then there is an interval $(c - \varepsilon, c + \varepsilon)$ on which $h(x) > 0$.

Hint #2: You may use the fact that if a continuous function h satisfies $h(x) > 0$ on an interval $[a, b]$, then $\int_a^b h(x)dx > 0$.