

MATH 16200 SECTION 50, HOMEWORK 1

DUE DATE THURSDAY JAN 18

Observe that if you are one of those folks who believes that reason and motivation is the Devil's device to fight against faith, then this problem set and in fact the whole course might not be your cup of tea. If you are not such a person, then please proceed to problem 1.

- (1) Is the set $\{x \in \mathbb{Q} : 1 \leq x \leq 2\}$ compact? Prove or disprove.
- (2) Prove that the union of each *finite* family of compact subsets of C is compact.
- (3) Show that a subset of \mathbb{Z} is compact iff it is finite.
- (4) A point m is called the **maximum** of a point set A iff the following two conditions hold.
 - (a) $m \in A$, and
 - (b) for all $x \in A$, $m \geq x$.

'Maximum' is the more standard terminology for what we have been calling a 'last point'. Show that if a is the maximum of A then $a = \sup A$. Is the converse true? Prove or disprove your answer.

The notation 'sup' stands for 'supremum', which is the Latin equivalent for the plain English 'least upper bound'. Frequently Latin and Greek words are used in mathematics instead of English ones, mostly for historical reasons.

- (5) In the rest of this problem set and the next we will guide you through the construction of a set C satisfying axioms 1, 2, and 3 which is 'essentially different' from the real numbers. In problems 5a and 5b we examine a set which is different, yet nevertheless 'essentially the same' (the technical term is 'isomorphic'). Then in problem 5c we prove a result which we will use in the next homework set to construct a C which is 'essentially different' from \mathbb{R} .
 - (a) Suppose that C is a set satisfying axioms 1 and 3 (such as the real numbers). Let a, b be points of C with $a < b$. Show that the region ab , considered as a set with the order induced from C , itself satisfies axioms 1, 2, and 3.
 - (b) Let C be the real numbers \mathbb{R} and let $a = 0, b = 1$. Write down a one-to-one correspondence between C and ab which **preserves order**, meaning that if x corresponds to $f(x)$ and y corresponds to $f(y)$, then $x < y$ if and only if $f(x) < f(y)$.
 - (c) Suppose that C is a set satisfying axioms 1 and 3, but which has a first point m and last point n . Show that by removing the first and last point, we obtain a set (namely, $C \setminus \{m, n\}$) which satisfies axioms 1, 2, and 3.
 - (d) Suppose instead that C merely satisfies axiom 1, and has a first and last point. Can we obtain a set satisfying axioms 1 and 2 by removing the first and last point?