The quantum dilogarithm and quantization

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Let S be a surface with punctures and G a split reductive group over $\mathbb{Q}$ with connected center, e.g. GL_n. We quantize the moduli space of G-local systems (a.k.a. the G-representation variety of S). This is a joint work with V.Fock.

The logic is this: a small modification of the moduli space of G-local systems on S has a cluster X-variety structure.

A cluster X-variety is a Poisson varietry. It is equipped with a collection of coordinate systems such that in each of the coordinate system $\{X_i\}$ the Poisson structure is

$$\{X_i, X_j\} = \varepsilon_{ij} X_i X_j, \quad \varepsilon_{ij} \in \mathbb{Z}$$

where the function ε_{ij} depend on the coordinate system. There is a group Γ_X acting by automorphisms of X. In the above example it contains the classical modular group of S.

Then we use the quantum dilogarithym to quantize an arbitrary cluster X-variety. The latter means the following data:

i) A series of infinite dimensional unitary projective representation of Γ_X in a Hilbert space H.

ii) A non-commutative deformation A of algebra of functions on X, with a Γ_X-action.

iii) A Γ_X-equivariant $*$-representation of the algebra A in (a subspace of) H.

Our construction can be viewed as a generalization of the Weil representation.