Lefschetz Theorems for the divisor class group

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The Lefschetz hyperplane theorem implies that over the complex number field $\mathbb{C}$, if Y is a smooth hypersurface section of a smooth projective variety X of dimension ≥ 4, then the restriction map on Picard groups $\mathrm{Pic}(X) \to \mathrm{Pic}(Y)$ is an isomorphism. When $\dim X = 3$, and Y is a "very general" hypersurface section which is "sufficiently positive", then the same conclusion holds; for the case $X = \mathbb{P}^3$, and Y of degree ≥ 4, this statement is the classical Noether-Lefschetz theorem. An algebraic treatement of these results may be given following Grothendieck (SGA2), using in particular his notion of the Lefschetz conditions for the formal completion of X along Y.

In this talk, I'll first review some of the classical material, and then discuss joint work with G. Ravindra, giving generalizations of the above results to the case when the variety X has normal singularities, and we work with the divisor class group in place of the Picard group. Our proofs are based on adapting Geothendieck's, and an elaboration of an unpublished proof of the Noether-Lefschetz theorem using deformation theory, due to Mohan Kumar and myself. I'll also sketch an application to a statement about 1-motives.