Test Final Math 17500

Name:

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Problem 6:

Problem 7:

Total

Problem 1

Find (if possible) all integers satisfying the following congruences simultaneously. Give the way you obtain the solution, not just the integer.

$$\begin{aligned} x &\equiv 1 \pmod{7} \\ x &\equiv 4 \pmod{9} \\ x &\equiv 3 \pmod{5}. \end{aligned}$$

Name:

Problem 2

1) Is 126 a quadratic residue mod89? Justify your answer.

2) Determine all primes p such that 7 is a quadratic residue $\bmod\, p$.

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Problem 3

Let x be real. Show that there is a reduced rational number $\frac{a}{b}$ with $0 < b \leq 20$ such that

$$|\xi - \frac{a}{b}| \leq \frac{1}{21b}.$$

Name:

Problem 4

1) Compute the continuous fraction expansions of the following numbers $\frac{19}{5}$, $\sqrt{5}$.

2) Evaluate the continued fraction expansions $[1, 2, 3, 4]$, $[1, 3, 1, 3, 1, 3, 1, 3, \cdots]$.

Name:

Problem 5

Let $\phi : \mathbf{N} \to \mathbf{N}$ be the Euler ϕ function.
Prove that there are infintely many integers n such that 3 does not divide $\phi(n)$.

Name:

Problem 6

1) Prove that if a has order h modulo a prime p and if h is even, then $a^{\frac{h}{2}} \equiv -1 \bmod p$.

2) Find a primitive root modulo 11. Does there exist a primitive root modulo 12? Modulo 8?

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Problem 7

Solve (if possible) the congruence. Find all solutions.

$$x^5 + x^4 + 1 = 0 \pmod{3^4}.$$