Math 199 Introduction to Analysis and Linear Algebra, Section 43

HOMEWORK # 8 , DUE FEBRUARY 28

Problem 1
Read Chapter 2.1, 2.2, 2.3. and 2.4. in *"Foundations of Mathematical Analysis"* by Paul J. Sally

Problem 2
Show that the following sets V are vector spaces over the field $\mathbb{R}$.

(1) The space of polynomials over $\mathbb{R}$ of degree less than n, $n \in \mathbb{N}$, i.e. $\mathbb{R}_{\leq n}[x] := \{a_0 + a_1 x + \cdots + a_n x^n \mid a_i \in \mathbb{R} \text{ for all } 0 \leq i \leq n\}$.
(2) $\mathbb{R}^n$.
(3) The space of functions from $\mathbb{R}$ to $\mathbb{R}$, i.e. $V = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is a function}\}$.

Problem 3

(1) Show that $\mathbb{R}^2 \times \{0\}$ is a vector subspace of $\mathbb{R}^3$.
(2) Show that $\mathbb{R}_{\leq k}[x]$ is a vector subspace of $\mathbb{R}_{\leq n}[x]$ for $k \leq n$.
(3) Let V be a vector space over $\mathbb{R}$. Let $\mathbf{v_1}, \cdots, \mathbf{v_m} \in V$ be a finite set of vectors. Show that $W = \text{span}\{\mathbf{v_1}, \cdots, \mathbf{v_m}\}$ is a vector subspace of V.
(4) **Bonus - not required** Show that the space of **continuous** functions from $\mathbb{R}$ to $\mathbb{R}$ is a vector subspace of the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$.

Problem 4
Consider the vector space $\mathbb{R}^4$ over $\mathbb{R}$. We will write an element of $\mathbb{R}^4$ as (x_1, x_2, x_3, x_4).

(1) Find a spanning set for $\mathbb{R}^4$ which contains the vectors $\mathbf{v_1} = (1,2,3,4)$ and $\mathbf{v_2} = (5,6,7,8)$.
(2) Find a basis for $\mathbb{R}^4$ (i.e. a spanning set of linear independent vectors), which contains the vectors $\mathbf{v_1} = (1,2,3,4)$ and $\mathbf{v_2} = (4,3,2,1)$. (Prove that the set of vectors you write down is indeed a basis).
(3) Consider now $\mathbb{R}^3$ as a vector space over $\mathbb{R}$. What is the span of the set $\{\mathbf{v_1} = (1,2,3), \mathbf{v_2} = (3,2,1)\}$? Can you interpret this geometricly?

Problem 5
Let V be a vector space over F. Let $\mathbf{v_1}, \mathbf{v_2} \in V$. Show that the set $\{\mathbf{v_1}, \mathbf{v_2}\}$ is linearly dependent if and only if there is some scalar $\alpha \in F$ such that $\mathbf{v_1} = \alpha \mathbf{v_2}$.