Math 201

Homework # 4, due January 31

Problem 1

Find first the general solution of the given differential equation, then find the solution which satisfies the given initial condition. Sketch the graph and describe the behavior of the solution as t goes to inftinity

(1) $y'' - 4y = 0$, $y(0) = 1$, $y'(0) = 0$
(2) $y'' + 4y' = 0$, $y(0) = 1$, $y'(0) = 1$
(3) $4y'' + y' - 2y = 0$, $y(0) = 1$, $y'(0) = 1$
(4) $y'' + 8y' - 9y = 0$, $y(1) = 1$, $y'(1) = 0$ (Problem 15 on page 142)
(5) $4y'' - y = 0$, $y(-2) = 1$, $y'(-2) = 1$ (Problem 16 on page 142).

Problem 2

Find a differential equation whose general solution is

(1) $y(t) = c_1 e^{\frac{1}{2}t} + c_2 e^{2t}$.
(2) $y(t) = c_1 e^{-2t} + c_2 e^{3t}$.
(3) $y(t) = c_1 e^{-2t} + c_2 e^{-\frac{1}{2}t}$. (Problem 18, p. 142)
(4) $y(t) = c_1 \cosh(t) + c_2 \sinh(t)$. (if you need a hint look at your notes from Monday, Jan 22 class or search in the book).

Problem 3

Find the solution to the initial value problem. Then determine the maximal/minimal value and the zeros of the solution $y(t)$ (if they exist!):

(1) $2y'' - 3y' + y = 0$, $y(0) = 2$, $y'(0) = \frac{1}{2}$ (Problem 20 on page 142)
(2) $y'' + 3y' + 2y = 0$, $y(0) = 1$, $y'(0) = 1$.

Problem 4

Compute the Wronksian $W_{y_1,y_2}(t)$ of the given pair of functions:

(1) $y_1(t) = -e^t \cos(t)$, $y_2(t) = -e^t \sin(t)$
(2) $y_1(t) = cos^2(t)$, $y_2(t) = 1 + cos(2t)$
(3) $y_1(t) = e^{2t}$, $y_2(t) = te^{2t}$
(4) $y_1(t) = e^{-t}$, $y_2(t) = 3e^{-t}$

Problem 5

Solve the following problems from the book:

(1) 7, 8, 9, 10 on page 151 - Justify your answer!
(2) Problems 21, 22 on page 151.
(3) Problems 23, 25, 26, 27 on page 152 - Justify your answer.