

PRACTICE MIDTERM

Problem 1

A tank initially contains 100 liter of pure water. A mixture containing a concentration of 4 gramm/liter of salt enters the tank at a rate of 2 liters/minute, and the well-stirred mixture leaves the tank at the same rate. Find the differential equation for the concentration of salt inside the tank at any time t . Find the limiting amount of salt in the tank as $t \rightarrow \infty$. Find the time (approximately) which elapses before the concentration of salt in the tank reaches 90% of its limiting value.

Problem 2

Solve the following initial value problems and describe the behaviour as t goes to infinity.

- (1) $ty' + 2y = 4t^2$, $y(1) = 2$.
- (2) $y' + 2y = te^{-2t}$, $y(1) = 0$.
- (3) $y'' + y' + 2y = 0$, $y(0) = 1$, $y'(0) = 0$.
- (4) $y'' + 4y' + 4y = 0$, $y(0) = 0$, $y'(0) = 1$.

Problem 3

Find the general solution (without initial conditions) of the following differential equations.

- (1) $y' = \frac{x^2}{y(1+x^3)}$
- (2) $(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$
- (3) $y'' - y' + 6y = 0$

Problem 4 Verify that the functions y_1, y_2 are solutions of the given differential equation. Do they constitute a fundamental set of solutions?

- (1) $y'' - 2y' + y = 0$, $y_1 = e^t$, $y_2(t) = 2te^t$.
- (2) $y'' + 4y$, $y_1(t) = 2 \cos(2t)$, $y_2(t) = \sin(2t)$.

Problem 5

Find the solution of the initial value problem

$$2y'' - 3y' + y = 0, y(0) = 2, y'(0) = \frac{1}{2}.$$

Determine the maximum value of the solution and find the t where the solution is 0.