Geometric Rigidity and Smooth Dynamics (MATH 35603 - 01)

Math 35603: Ergodic Theory and Smooth Dynamics-2
MWF 12:30PM- 1:20PM, Ryerson 276

The goal of this course is to introduce methods from smooth ergodic theory and dynamical systems in their application to rigidity questions in Riemannian geometry. The results I will discuss illustrate the beautiful interplay between dynamics and geometry, with each area motivating questions and results in the other.

There are no prerequisites for the course, and we will take as a black box some of the results presented last quarter, in particular, ergodicity of the geodesic flow in negative curvature. The course should be accessible to all graduate students who are comfortable with the basics of Romanian geometry. The central topic I will present is:

Marked Length Spectrum Rigidity: Let M be a closed Riemannian manifold whose sectional curvatures are all negative. In each free homotopy class of curves in M, there is a unique closed geodesic, and so one can define a marked length spectrum function which assigns to the each class $[\gamma]$ the length $\ell(\gamma)$ of this closed geodesic. In the spirit of classical inverse spectral problems, it is natural to ask whether the marked length spectrum determines the metric. The answer to this question remains open in general, but has been solved completely for surfaces by J.-P. Otal and (independently slightly later, but in greater generality) C. Croke. In this course, Ill explain a proof of this marked length spectrum rigidity for negatively curved surfaces:

Theorem (Otal): *Let S and S be closed, negatively curved surfaces with the same marked length spectrum. Then S is isometric to S.*

An expository paper presenting this result can be found at
http://www.math.uchicago.edu/ wilkinso/papers/PCMI.pdf .
Time permitting, I will also discuss:

- the proof, due to D. Burago and S. Ivanov, of the “Hopf Conjecture”: any Riemannian metric on a torus without conjugate points is flat.
- a remarkable rigidity result of G. Besson–G. Courtois–S. Gallot: If a negatively curved metric on a locally symmetric compact manifold has the same volume and entropy as its locally symmetric counterpart, then the two Riemann structures are isometric.