AMIE WILKINSON – RESEARCH INTERESTS

My research lies in the area of smooth dynamical systems and is concerned with the interplay between dynamics and other structures in pure mathematics – geometric, statistical, topological and algebraic.

The broad scope of dynamical methods and applications can be traced to its origins. The field of dynamical systems was pioneered around the turn of the 20th Century by Henri Poincaré and George Birkhoff, who attempted to attach a rigorous mathematical framework to natural questions arising in physics (in particular, the statistical properties of ideal gases and the stability of planetary motion). Other early contributors to the field were Eberhard Hopf, who used dynamics to describe the statistical behavior of geodesics on negatively curved surfaces, John von Neumann, who laid the foundations of Ergodic Theory, an analytic and measure-theoretic approach to dynamics, and Andrey Kolmogorov, who created the modern field of dynamics as we know it today.

In simple terms, a dynamical system is a space with a set of rules (a transformation) that can be iterated. Each iteration represents the passing of one unit of time, and one asks what phenomena can be observed under long-term iteration. A more modern definition of dynamical system replaces the single transformation by the action of an infinite group or semigroup. In smooth dynamics, the action of this group is by smooth transformations, such as diffeomorphisms or flows given by a smooth vector field. I am interested in the following broad questions in smooth dynamics:

1. *What are the mechanisms for chaotic behavior?* By mechanism, I mean a coarse geometric and/ or topological property of the system that can be verified in practice and is robust under perturbations of the system. Chaotic can mean several things depending on the context, but a common property of chaotic systems is mixing: arbitrary subsets of the system will evenly intertwine over time. In my own work, I have extensively studied the mechanism of partial hyperbolicity and how it produces stable mixing. The notion of mechanism can be weakened to allow for phenomena that appear for a "typical system," but not robustly. In recent work with Artur Avila and Sylvain Crovisier, I've proved a formulation of the ergodic hypothesis for systems with positive entropy: in the C^1 topology, ergodicity is generic (i.e. holds for a residual set of conservative diffeomorphisms). The generic mechanism here is nonvanishing Lyapunov exponents, invariants that detect infinitesimal long-term expansion and contraction rates.

2. *What geometric properties of a Riemannian manifold produce chaotic distribution of its geodesics?* A dynamical system associated to any Riemannian manifold is the geodesic flow, first studied by Morse, Hadamard, Hedlund and E. Hopf. Dynamical properties of the geodesic flow capture the geometric properties of geodesics, including the distribution of closed geodesics and the typical behaviors of infinite geodesics. If the manifold is compact and negatively curved, then the geodesic flow is chaotic, as measured by any reasonable standard (the list of names behind these assertions is huge, going back to the pioneers and including works as recent as the early 2000s). When either negative curvature or compactness is relaxed, the dyanmics can change radically. I have studied the geodesic flow for certain incomplete, negatively curved manifolds carrying the so-called Weil-Petersson metric from Teichmueller theory.

3. *Which infinite groups can act smoothly on a manifold, and under what conditions can such actions be deformed?* Zimmer initiated an ambitious program to classify actions of "large" infinite groups, such as lattices in semisimple Lie groups. Typically, such groups cannot act, or actions when they do exist are quite rigid (i.e., they cannot be deformed). I've studied actions of groups that are "smaller" in some sense (for example, discrete solvable groups) and can be deformed smoothly, but only in certain prescribed ways. These actions parallel in many ways the properties of partially hyperbolic diffeomorphisms: flexible enough to produce a rich class of examples, but rigid enough to admit some level of classification.

4. *What features of a smooth dynamical system are rigid, and how flexible are invariants under perturbation?* Continuing with the theme of flexibility and rigidity, I am interested in how much information about a dynamical system is needed to determine the system (up to a reasonable form of equivalence). There are several "soft" invariants, such as entropy, Lyapunov exponents, and dimension of invariant sets, which taken alone can be perturbed freely, but cannot be perturbed independently. Perturbing Lyapunov exponents can produce desirable chaotic behavior, which I have exploited in my work on stable mixing. On the other hand, Lyapunov exponents of certain classes of systems such as geodesic flows cannot be perturbed in certain directions and can completely determine the manifold up to isometry (as has been shown in recent work by my student Clark Butler).