**On De Giorgi conjecture in dimensions $N \geq 9$**

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**Abstract:** A celebrated conjecture due to De Giorgi states that any bounded solution of the equation $\Delta u + (1 - u^2)u = 0$ in $\mathbb{R}^N$ with $\frac{\partial u}{\partial y_N} > 0$ must be such that its level sets $\{u = \lambda\}$ are all hyperplanes, **at least** for dimension $N \leq 8$. A counterexample for $N \geq 9$ has long been believed to exist. Based on a minimal graph $\Gamma$ which is not a hyperplane, found by Bombieri, De Giorgi and Giusti in $\mathbb{R}^N$, $N \geq 9$, we prove that for any small $\alpha > 0$ there is a bounded solution $u_\alpha(y)$ with $\frac{\partial u_\alpha}{\partial y_N} > 0$, which resembles $\tanh\left(\frac{t}{\sqrt{2}}\right)$, where $t = t(y)$ denotes a choice of signed distance to the blown-up minimal graph $\Gamma_\alpha := \alpha^{-1}\Gamma$. This solution constitutes a counterexample to De Giorgi conjecture for $N \geq 9$.