You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed with one exception: you may use a clean copy of the Rules of Arithmetic for the Real Numbers.

Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

Finally, there is partial credit to be awarded, so please SHOW YOUR WORK!

1. **Solving Equations** (24 points, 6 each) Solve each of the following for the real variable $x$.

(a) \[ \frac{2}{x} = \frac{1}{1 + x} \]
(b) \[ e^x(5 + 10x) = 0 \]
(c) \[ x^4 + 5x^3 + 5x^2 - 5x - 6 = 0 \] (Hint: Try $x = \pm 1$.)
(d) \[ |3x + 12| = x + 8 \]

2. **Solving Inequalities** (24 points, 8 each) Solve each of the following for the real variable $x$. In each case, express your answer in interval notation, and graph your solution on a number line.

(a) \[ 3x + 5(x - 2) \geq -2x + 15 \]
(b) \[ \frac{3}{x+3} \leq x + 1 \]
(c) \[ (x + 5)^2(x - 6)(4 - 2x) > 0 \]

3. **Three lines** (21 points, 7 each) Let $A = (-3, -7)$ and $B = (4, -2)$ be points in the $xy$-plane.

(a) Find the equation of the line $\ell_1$ through $A$ and $B$.
(b) Find the equation of the line $\ell_2$ that is parallel to $\ell_1$ and passes through $P = (-1, 3)$.
(c) Find the equation of the line $\ell_3$ that is perpendicular to $\ell_1$ and passes through $Q = (10, 0)$.

4. **A parabola.** (24 points, 6 each)

Consider the parabola given by the equation $y = -\frac{1}{9}(x + 3)(x + 6)$.

(a) Identify the vertex of the first parabola.
(b) Write the equation of the parabola in Vertex Form.
(c) Identify the $x$- and $y$-intercepts of the parabola.
(d) Graph the first parabola on a well-labelled set of axes.
5. A circle. (16 points, 8 each) Consider the circle given by the equation $x^2 + 5x + y^2 - 7y - \frac{13}{2} = 0$.

(a) Find the center and radius of the circle. (Hint: Complete the square–twice!)

(b) Find the two points of intersection of the circle with the line $y = \frac{1}{3}x + 1$. (Hint: $\sqrt{2196} = 36$.)

6. Factorization and Algebraic Expressions. (12 points, 8/4)
Consider the polynomial function $p(x) = x^4 - 81$.

(a) Use the “difference of squares” technique to factor $p(x)$ as completely as possible (over the real numbers).

(b) Find all values of $x$ such that $p(x) = 0$.

7. Sequences. (16 points, 8 each) Consider the sequence $4, -2, \ldots$

(a) If the sequence is arithmetic (constant difference between consecutive terms), find the tenth term.

(b) If the sequence is geometric (constant ratio between consecutive terms), find the tenth term.

8. A logarithmic function. (16 points, 4 each) Consider the equation $f(x) = \log_2 \left( \frac{1}{2 + x^2} \right)$.

(a) Find $f(0)$.

(b) Find the value(s) of $a$ such that $f(a) = -3$.

(c) Show that $\frac{1}{2 + x^2} \leq \frac{1}{2}$ for every real number $x$.

(d) Show that $f(x) \leq -1$ for every real number $x$.

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