You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed with one exception: you may use a clean copy of the Rules of Arithmetic for the Real Numbers.

Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

Finally, there is partial credit to be awarded, so please SHOW YOUR WORK!

1. **Points and Lines** (18 points, 6 each) Let $A = (-4, 1)$ and $B = (5, -4)$.
   
   (a) Find the equation of the line containing $A$ and $B$.
   
   (b) Find the point on the line that has $x$-coordinate 7.
   
   (c) Through which three quadrants of the $xy$-plane does the line pass? Explain.

2. **Solving Inequalities** (24 points, 8 each) Solve each of the following for the real variable $x$. In each case, express your answer in interval notation, and graph your solution on a number line.
   
   (a) $2x + 4 \leq 8$
   
   (b) $|2x + 4| \leq 8$
   
   (c) $|2x + 4| \leq x + 8$

3. **Two Lines** (18 points, 8/5/5) Let $k$ be a constant, and consider the two lines:
   
   $\ell_1 : -2x + 4y + 7 = 0$
   
   $\ell_2 : kx - 6y + 1 = 0$
   
   (a) If $k = 1$, find the point of intersection of these two lines.
   
   (b) Find the value(s) of $k$ so that $\ell_1$ and $\ell_2$ are parallel.
   
   (c) Find the value(s) of $k$ so that $\ell_1$ and $\ell_2$ are perpendicular.

4. **Functions and Domains** (30 points, 5 each) Consider the function
   
   $f(x) = \frac{(x - 2)^2(x + 3)}{(x - 2)(8x - 4)(x + 5)}$
   
   (a) Find the domain of $f$.
   
   (b) Identify all $x$-intercepts of $y = f(x)$.
   
   (c) Identify the $y$-intercept of $y = f(x)$.
   
   (d) Identify any horizontal asymptotes of $y = f(x)$.
   
   (e) Identify any vertical asymptotes of $y = f(x)$.
   
   (f) Identify all intervals on which $f$ is positive.
5. **A Parabola.** (24 points, 8 each)

A golf ball in flight follows a roughly parabolic arc. Suppose that the height of the ball is given by the equation \( y = 144t - 16t^2 \) while it is in flight, where \( y \) is measured in feet and \( t \) is measured in seconds.

(a) When does the ball land? (Assume it is struck when \( t = 0 \)).
(b) Find the maximum height of the ball while it is in flight.
(c) Graph the height function on a well-labelled set of axes.

6. **Equations and Graphs.** (16 points, 8 each) Consider the two simultaneous equations:

\[ y = \sqrt{2x + 5} \quad \text{and} \quad y = 5 - x \]

(a) Find the solution(s) to this pair of simultaneous equations.
(b) Graph the two equations on a common set of axes, and be sure to indicate the point(s) of intersection.

7. **Algebraic Expressions.** (16 points, 4/4/8)

Let \( a, b > 0 \). The **arithmetic mean** of these two numbers is defined to be: \( \frac{a + b}{2} \).

The **harmonic mean** of these two numbers is defined to be: \( \frac{2}{\frac{a}{b} + \frac{b}{a}} \).

(a) Find the arithmetic mean of 2 and 4.
(b) Find the harmonic mean of 2 and 4.
(c) Show that for any \( a, b > 0 \), the harmonic mean of \( a \) and \( b \) is less than or equal to the arithmetic mean of \( a \) and \( b \).

8. **Logarithms and Exponentials.** (16 points, 8 each)

(a) Evaluate each of the following expressions: \( \log_2(0.25), \ 2^{-3}, \ (0.01)^{1/2}, \ \frac{10^{-6}}{10^{-8}} \)
(b) If \( a = \log_b x \), find each of the following (in terms of \( a \)): \( \log_b(x^2), \ \log_b(\frac{1}{x}), \ \log_{(b^2)} x \)

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