You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

1. **Inequalities** (21 points, 7 each)
   (a) Solve the inequality $-\frac{1}{4}x + 4 \geq 0$, and express your answer in interval notation.
   (b) Solve the inequality $12x + 36 < 0$, and express your answer in interval notation.
   (c) Solve the inequality $\frac{-\frac{1}{2}x + 4}{12x + 36} < 0$, and express your answer in interval notation.

2. **Equations and Graphing** (26 points, 8/10/8)
   (a) Graph the line $x + 2y - 2 = 0$, being sure to indicate the $x$- and $y$-intercepts and the slope.
   (b) Graph the parabola $y = -\frac{1}{4}x^2 + 4x - 13$, being sure to indicate the $x$- and $y$-intercepts and the vertex.
   (c) Find the points of intersection of the line from part (a) and the parabola from part (b).

3. **Absolute Value** (20 points, 10 each)
   (a) Solve the inequality $|5 - 7x| \geq 2$, and graph your solution on the number line.
   (b) Solve the equation $-2 + |3x - 8| = x^2$.

4. **Linear Equations.** (20 points, 5 each)
   Consider the two lines below:
   
   Line 1: $x + 3y - 3 = 0$
   Line 2: $-4x + 9y - 4 = 0$

   (a) Find the point of intersection of Line 1 and Line 2.
   (b) Are the two lines perpendicular? Explain.
   (c) Find the equation of the line parallel to Line 2 that passes through the $x$-intercept of Line 1.
   (d) Graph Line 1, Line 2, and the line from part (c) on one set of axes.

5. **The Sum of Cubes.** (18 points, 10/8)
   (a) Use the rules of algebra to verify the formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.
   (b) Noting that $1729 = 10^3 + 9^3$ and that $1729 = 12^3 + 1^3$, use the formula from part (a) to find the prime factorization of the number 1729.
6. **An Equation with Radicals.** (24 points, 6 each) Consider the equation in one variable $x$ given by

$$8 + 7\sqrt{x} = x.$$

(a) Substitute $x = y^2$ to write the equation as a polynomial in the variable $y$.
(b) Solve the equation from part (a) for the variable $y$.
(c) Solve the original equation for the variable $x$.
(d) Check your answer(s) from part (c) in the original equation and explain.

7. **Exponentials and Logarithms.** (20 points, 10 each)

(a) Solve the logarithmic equation: $\log_3(x + 2) - \log_3(x + 1) = 2$.

(b) Put the following five real numbers in increasing order:

$$\frac{1}{25}, \sqrt[3]{-\frac{1}{27}}, \log_{10}\left(\frac{1}{100}\right), 4^{-3}, \sqrt{11}$$

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