You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

1. **Lines and the Coordinate Plane** (28 points, 7 each) Let $A = (-1, 1)$, $B = (5, 3)$, and $C = (4, -2)$.
   
   (a) Find the equation of the line through $A$ and $B$.
   (b) Find the equation of the horizontal line through $C$.
   (c) Find the point of intersection of the two lines from part (a) and (b).
   (d) Find a point $D$ so that the four points $A$, $B$, $C$, and $D$ form the vertices of a parallelogram.

2. **Functions and Domains** (24 points, 6 each) Consider the functions

   $$f(x) = 3x^{3/2} - 10x^{1/2} + 3x^{-1/2}$$

   $$g(x) = \frac{1}{\sqrt{16 - x^2}}$$

   (a) Find the domain of $f$.
   (b) Identify all $x$-intercepts of $y = f(x)$.
   (c) Find the domain of $g$.
   (d) Identify all $x$-intercepts of $y = g(x)$.

3. **Parabolas.** (24 points, 6 each)
   Consider the two parabolas:

   $$y = -x^2 + 5x - 3$$

   $$y = x^2 + kx + 8$$

   (a) Identify the vertex of the first parabola.
   (b) Identify the $x$-intercepts of the first parabola.
   (c) Graph the first parabola.
   (d) For what value of $k$ do the two parabolas intersect at exactly one point?

4. **Linear Equations.** (22 points, 6/6/10)
   Consider the three lines below:

   $$\text{Line 1: } -4x + 12y - 3 = 0$$

   $$\text{Line 2: } 6x + 2y - 1 = 0$$

   $$\text{Line 3: } y = -3(x + 4)$$

   (a) Are any two of the lines parallel? Explain.
   (b) Find the point of intersection (if any) of Line 1 and Line 2.
   (c) Graph all three lines on one set of axes.
5. **Binomials and Algebraic Expressions.** (16 points, 8 each)

   (a) Expand the expression \((a - 2b)^3\) by multiplying out and collecting terms.

   (b) Evaluate the expression when \(a = 5\) and \(b = -1\).

6. **Polynomials and Rational Functions.** (24 points, 8 each) Consider the function 

   \[ y = \frac{2x^2 - 5x - 18}{x - 4}. \]

   (a) Use long division of polynomials to identify polynomials \(q(x)\) (the quotient) and \(r(x)\) (the remainder) such that 
   \[ 2x^2 - 5x - 18 = q(x) \cdot (x - 4) + r(x). \]

   (b) Identify any vertical asymptotes of the function.

   (c) Identify any other asymptotes of the function.

7. **Exponentials and Logarithms.** (20 points, 10 each)

   (a) Solve the logarithmic equation: 
   \[ \frac{1}{2} \log_4(5x + 1) = 1. \]

   (b) Use the substitution \(u = e^{2x}\) to help solve the equation 
   \(e^{4x} - 3e^{2x} + 2 = 0.\)

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