(5) (a) \[ f(x) = x^3 - 6x^2 + 9x + 2 \]
\[ f'(x) = 3x^2 - 12x + 9 \]
\[ = 3(x^2 - 4x + 3) \]
\[ = 3(x-1)(x-3) \]

Crit. Pts.
I. \( x = 1, 3 \)
II. None
III. \( x = 0, 4 \)

\[ \begin{array}{c|c|c}
\text{Interval} & f' & f \\
\hline
[0,1) & + & \text{inc.} \\
(1,3) & - & \text{dec.} \\
(3,4] & + & \text{inc.} \\
\hline
\end{array} \]

Therefore, \( x = 1 \) is a local max.
\( x = 3 \) is a local min.
by First Deriv. Test.

Since no global max/min exist, we compare:
\[ f(0) = 2 \]
\[ f(1) = 6 \]
\[ f(3) = 2 \]
\[ f(4) = 6 \]

All 4 local max/min are also global max/min.

(b) If \( y = ax^3 + bx^2 + cx + d \),
then \( y' = 3ax^2 + 2bx + c \)
By Q. F., this has at most two crit. pts., namely
when \( y' = 0 \), that is,
\[ x = \frac{-2b \pm \sqrt{(2b)^2 - 4(3a)c}}{2(3a)} \]

This, \( \mathbb{R} \) is divided into three intervals on which it is
either increasing or decreasing.
Since an increasing function may only take on the value 0 once,
there are at most three places where \( y = 0 \).
\( f(x) = x^2 \cdot e^{-x} \)

(a) \[ f'(x) = x^2 \cdot e^{-x} \cdot (-1) + 2x \cdot e^{-x} \]
\[ = (2x - x^2) \cdot e^{-x} \]

(b) \[ f''(x) = (2x - x^2) \cdot e^{-x} \cdot (-1) + (2 - 2x) \cdot e^{-x} \]
\[ = e^{-x} \cdot (x^2 - 4x + 2) \]

(c) \[ \text{Crit. pts. I. } f'(x) = 0 \text{ when } x = 0, 2 \]
\[ \text{II. } f' \text{ always defined} \]
\[ \text{III. } \text{No given endpoints} \]

(d) \[ \begin{array}{c|c|c}
\text{Inc/Dec} & f' & f \\
\infty, 0) & - & \text{decreasing} \\
(0, 2) & + & \text{increasing} \\
(2, \infty) & - & \text{decreasing} \\
\end{array} \]

(e) \( x = 0 \) is a local min.
\( x = 2 \) is a local max.

(f) Possible Inflection: I. \( e^{-x} \neq 0 \)
\[ x^2 - 4x + 2 = 0 \text{ when } x = \frac{4 \pm \sqrt{16-8}}{2} = 2 \pm \sqrt{2} \]
\[ \text{II. } \text{None.} \]

\[ \begin{array}{c|c|c}
\text{Up/Down} & f'' & f \\
(-\infty, 2-\sqrt{2}) & + & \text{up} \\
(2-\sqrt{2}, 2+\sqrt{2}) & - & \text{down} \\
(2+\sqrt{2}, \infty) & + & \text{up} \\
\end{array} \]

(g) \( x = 2 \pm \sqrt{2} \) are both inflection pts.

(h) No vertical asymptotes
Horiz. asymptote \( y = 0 \)
because \( \lim_{x \to \infty} f(x) = 0 \)

\( f(0) = 0 \)
\( f(2) = \frac{4}{e^2} \approx 0.5 \)