Calculus Placement Exam
Harris School of Public Policy
September 23, 2013

You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted. Finally, there is partial credit to be awarded, so please SHOW YOUR WORK!

1. **Limits I** (24 points, 6 each)
   Evaluate the following limits:
   
   (a) \( \lim_{x \to 4} \frac{x^2 - 16}{x^2 - 7x + 12} \)
   
   (b) \( \lim_{x \to 2} \frac{\sqrt{x + 1} - \sqrt{3}}{x - 2} \)
   
   (c) \( \lim_{x \to -\infty} \frac{1 + e^{-x}}{1 - x} \)
   
   (d) \( \lim_{x \to -1} e^{x+1}(4 - x^2) \)

2. **Limits II** (16 points, 8 each)
   
   (a) Evaluate the following limit: \( \lim_{x \to a} \frac{x^4 - a^4}{x - a} \)
   
   (b) What is the natural context for the limit you computed above? Explain what this limit represents in the context of Calculus.

3. **Differentiation.** (24 points, 8 each)
   Differentiate the following functions. You may use any theorems.
   
   (a) \( h(x) = x^3 \cdot \ln(1 + \sqrt{x}) \)
   
   (b) \( j(x) = \left( \frac{1 - 3x}{1 + 3x} \right)^{1/3} \)
   
   (c) \( k(x) = 2^{-x^2+3x} \)

4. **Asymptotes.** (12 points)
   Find all asymptotes for the function \( f(x) = \frac{(3x - 6)(x + 6)}{(x + 4)(x + 10)} \), and write down the limits that justify the existence of these asymptotes.

5. **Optimization.** (20 points)
   Find all global and local maxima and minima of the function \( f(x) = x^3 + 6x^2 - 15x - 72 \) on the interval \([-10, 3]\).
6. Analysis of Functions I (25 points, 5/5/15)

Consider the following data about a function $f : \mathbb{R} \rightarrow \mathbb{R}$.

Some Values: $f(2) = 0$, $f(4) = 0$, $f(8) = 0$, $f(0) = 5$, $f(-2) = 6$, and $f(6) = -2$

Some Limits: $\lim_{x \to -\infty} f(x) = 3$, $\lim_{x \to +\infty} f(x) = +\infty$, $\lim_{x \to -3^-} f(x) = -\infty$, and $\lim_{x \to -3^+} f(x) = +\infty$

Derivative Data:

<table>
<thead>
<tr>
<th>Interval</th>
<th>$f'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-\infty, -2]$</td>
<td>$+$</td>
</tr>
<tr>
<td>$(-2, 3)$</td>
<td>$-$</td>
</tr>
<tr>
<td>$(3, 6)$</td>
<td>$-$</td>
</tr>
<tr>
<td>$(6, +\infty)$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

Second Derivative Data:

<table>
<thead>
<tr>
<th>Interval</th>
<th>$f''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-\infty, -4)$</td>
<td>$+$</td>
</tr>
<tr>
<td>$(-4, 3)$</td>
<td>$-$</td>
</tr>
<tr>
<td>$(3, +\infty)$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

(a) Identify any local maxima and minima of $f$.
(b) Identify any inflection points of $f$.
(c) Make an accurate graph of $y = f(x)$ on an appropriately scaled set of axes. Make sure the graph illustrates all of the indicated behavior.

7. Analysis of Functions II (33 points, 3 each, except (j) which is 6)

Let $g(x) = (12 - 2x)^{3/2}$.

(a) Compute $g'(x)$.
(b) Identify all critical points of $g$.
(c) Identify the intervals on which $g$ is increasing and decreasing.
(d) Identify all local maxima and minima of $g$.
(e) Compute $g''(x)$.
(f) Identify all possible inflection points of $g$.
(g) Identify the intervals on which $g$ is concave up and concave down.
(h) Identify the inflection points of $g$.
(i) Identify any asymptotes of $g$.
(j) Make an accurate graph of $y = g(x)$ on an appropriately scaled set of axes.

8. Partial Derivatives (20 points, 8/8/4)

Let $f(x, y) = x^2 + xy^3 - 6y$.

(a) Compute $\frac{\partial f}{\partial x}$.
(b) Compute $\frac{\partial f}{\partial y}$.
(c) If $y = 2$, find $x$ such that $\frac{\partial f}{\partial x} = 0$. 

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